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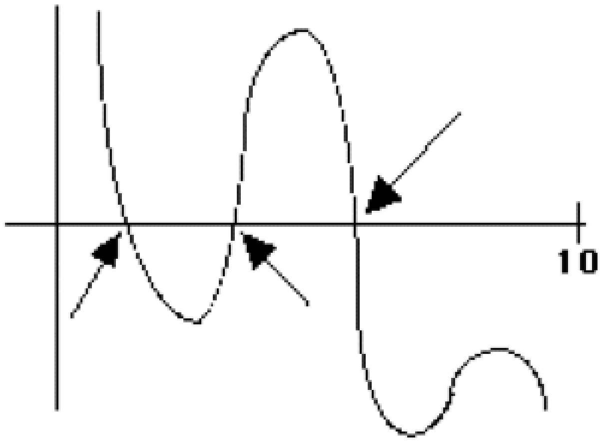
76. A B is true as in every neighborhood of x = a $f(a)$ is greater than any other $f(x)$. C is true because $f(a)$ exists. D is true because both sides of graph converge on the same point at x = a. E is true because, again, both sides of graph converge on the same point at x = a. $\boxed{A}$ is false because $\lim_{x \to a} f(x) \neq f(a)$.

77. C For the tangent lines to be parallel,
$f'(x) = g'(x) \quad f'(x) = 6e^{2x} \quad g'(x) = 18x^2 \to 6e^{2x} = 18x^2 \to 6e^{2x} - 18x^2 = 0$
Solving on calculator yields $\boxed{x = -.391}$

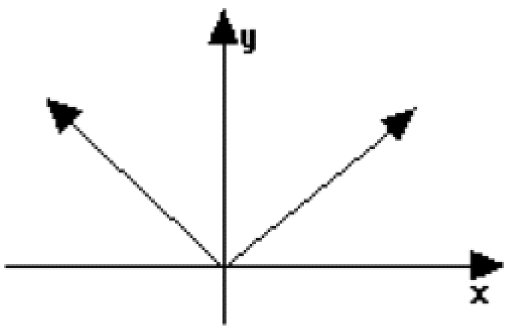
78. B $A = \pi r^2 \to \dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt}$ But, $C = 2\pi r \to \dfrac{dA}{dt} = C\dfrac{dr}{dt} = \boxed{.-1C}$

79. A For a function to have a relative maximum, the 1st derivative must equal 0 and the 2nd derivative must be negative, meaning the 1st derivative must be decreasing. On a < x < b, g'(x) is increasing when it is 0, h'(x) is not zero. That means that $\boxed{\text{only } f(x)}$ has the required relative maximum.

80. B Critical points occur where the 1st derivative is 0. Graphing the function on the given interval



yields $\boxed{\text{3 such points}}$.



81. D II At x = 0, the $f(0)$ is less than all other $f(x)$, so it is an absolute maximum. III $\lim_{x \to 0-} f'(x) = \lim_{x \to 0} f'(x)$ so f is not differentiable. $\boxed{I}$ The $\lim_{x \to 0-} f(x) = \lim_{x \to 0} f(x) = f(0)$ so $f(x)$ is continuous at x = 0.

82. E 1st Fundamental Theorem of Calculus.

$$\frac{d}{dx}(F(2x)) = F'(2x)(2) = 2f'(2x) \rightarrow \int f'(2x)dx = \frac{1}{2}F(2x) + C \rightarrow$$

$$\int_1^3 (f(2x))dx = \frac{1}{2}[F(2(3)) - F(2(1))] = \boxed{\frac{1}{2}F(6) - \frac{1}{2}F(2)}$$

83. B $\lim_{x \to a} \frac{(x^2 - a^2)}{(x^2 - a^2)(x^2 + a^2)} = \lim_{x \to a} \frac{1}{(x^2 + a^2)} = \boxed{\frac{1}{2a^2}}$

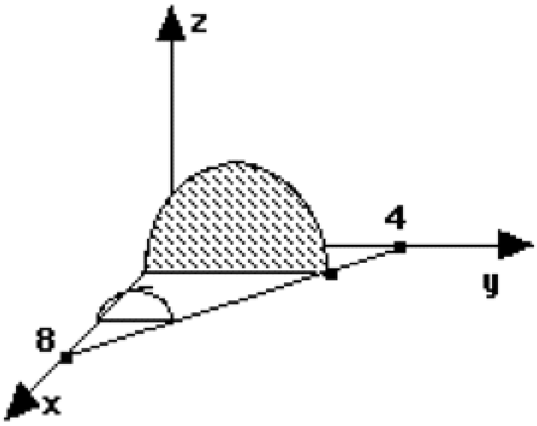
84. A $\frac{dy}{y} = k\ dx \rightarrow \int \frac{dy}{y} = \int k\ dx \rightarrow \ln y = kx + C$ Use point $(0, y_0)$ to find C, then point $(10, 2y_0)$ to find k. $\ln(y_0) = k(0) + C \rightarrow C = \ln(y_0)$

$$\ln(2y_0) = 10k + \ln(y_0) \rightarrow \ln(2y_0) - \ln(y_0) = 10k \rightarrow \ln\left(\frac{2y_0}{y_0}\right) = 10k \rightarrow k = \frac{\ln 2}{10} =$$

$\boxed{0.069}$

85. C Area = $\frac{1}{2}(3)(10+30) + \frac{1}{2}(2)(30+40) + \frac{1}{2}(1)(40+20) = 60 + 70 + 30 = \boxed{160}$.

86. C



$x + 2y = 8 \rightarrow y = 4 - \frac{1}{2}x \rightarrow r = 2 - \frac{1}{4}x$ $V = \frac{1}{2}\pi \int_0^8 \left(2 - \frac{1}{4}x\right)^2 dx = \boxed{16.755}$

87. D $f'(x) = 4x^3 + 4x \rightarrow f'(x) = 1 = 4x^3 + 4x \rightarrow 4x^3 + 4x - 1 = 0$ Using calculator to solve, $x = .237$ $f(.237) = .115 \rightarrow y - .115 = 1(x - .237) \rightarrow \boxed{y = x - .122}$

88. C $u = \ln x \quad du = \dfrac{dx}{x} \rightarrow F(x) = \left(\dfrac{(\ln x)^4}{4} \right) + C \rightarrow$

$F(1) = \left(\dfrac{(\ln 1)^4}{4} \right) + C \rightarrow 0 = 0 + C \rightarrow C = 0 \rightarrow F(9) = \left(\dfrac{(\ln 9)^4}{4} \right) = \boxed{5.827}$

89. B $f'(x) = 0 = \left(x^2 - 4\right) g(x) \; with \; g(x) < 0 \rightarrow x = \{-2, 2\}$

$f''(x) = \left(x^2 - 4\right) g'(x) + 2x \; g(x) \rightarrow f''(-2) = 0 - 4g(x) \; and \; f''(2) = 0 + 4g(x)$

Since $g(x) < 0 \rightarrow f''(-2) > 0 \; and \; f''(2) < 0 \rightarrow f'(-2) = 0, f''(-2) > 0 \rightarrow$

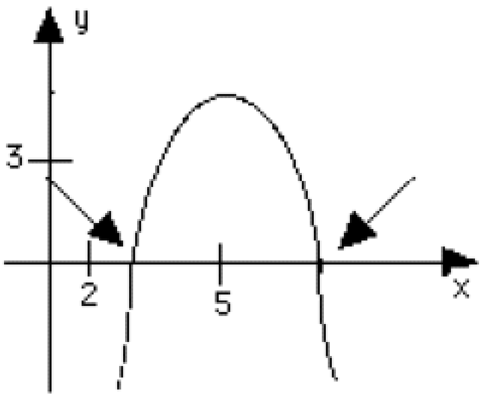
$\boxed{\text{relative minimum at } x = -2}$ and

$\boxed{f'(2) = 0, f''(2) < 0 \rightarrow \text{relative maximum at x=2}}$

90. D $A_{\triangle} = \dfrac{1}{2} bh \rightarrow \dfrac{dA_{\triangle}}{dt} = \dfrac{1}{2}\left(b\dfrac{dh}{dt} + h\dfrac{db}{dt} \right) = \dfrac{1}{2}\left(-3b + 3h\right)$ If

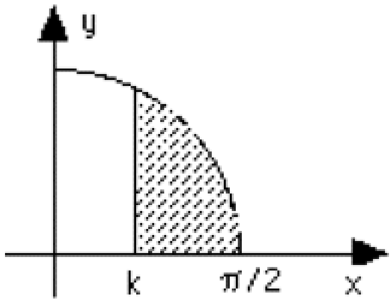
$b > h \rightarrow \dfrac{dA_{\triangle}}{dt} < 0 \quad b = h \rightarrow \dfrac{dA_{\triangle}}{dt} = 0$ 1st derivative is negative means function is decreasing. 1st derivative is positive means the function is increasing. This means that $\boxed{\text{the function is decreasing when } b > h}$

91. E Graph it.



One can see that it has $\boxed{I}$ a horizontal tangent, it $\boxed{II}$ has 2 zeroes, and by the intermediate value theorem, there $\boxed{III}$ there must be a c, 2 < c < 5 such that, $c = 3$, since $-5 < f(x) < 5$ on the interval 2 < c < 5.

92. D



$\int_k^{\pi/2} (\cos x) dx = .1 \rightarrow \sin\left(\dfrac{\pi}{2}\right) - \sin k = .1 \rightarrow \sin k = \sin\left(\dfrac{\pi}{2}\right) - .1 \rightarrow \sin k = .9 \rightarrow \boxed{k = 1.120}$

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