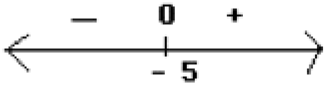
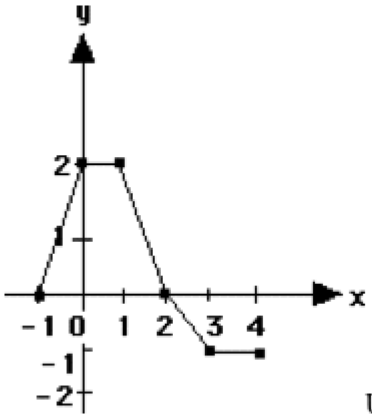
Return to Mr Calculus

1. D $y' = x^2 + 10x \rightarrow y'' = 2x + 10$ Inflection points happen when second



derivative is 0. $2x + 10 = 0 \rightarrow x = \boxed{-5}$ Second derivative is negative for x < –5 and positive for x > –5 so it is an inflection point.



2. B Use areas to get integral. From –1 to 2 area is a trapezoid, area $\frac{1}{2}(2)(3+1) = 4$ and from 2 to 4 is a trapezoid $\frac{1}{2}(-1)(2+1) = -1.5$ so answer is $\boxed{2.5}$

3. C $\int_1^2 \left(x^{-2}\right)dx = \left(\frac{x^{-1}}{-1}\right)\Big]_1^2 = \left(\frac{-1}{x}\right)\Big]_1^2 = -\frac{1}{2} - -1 = \boxed{\frac{1}{2}}$

4. B A gives the conditions for the Mean Value Theorem. B This is Rollés Theorem which has the added condition that $f(a) = f(b) = 0$, so this is false. The conditions given make C and D true—absolute maximum and minimum. E The conditions are what are necessary for the integral to exist.

5. E $-\cos t\Big]_0^x = -\cos x - -\cos 0 = \boxed{1 - \cos x}$

6. A $2^2 + 2y = 10 \rightarrow y = 3$ Take derivative implicitly $2x + x\frac{dy}{dx} + y = 0$. Solve for $\frac{dy}{dx}$ $x\frac{dy}{dx} = -(2x + y) \rightarrow \frac{dy}{dx} = \frac{-(2x+y)}{x}$ At (2,3). $\frac{dy}{dx} = \frac{-(2(2)+3)}{2} = \boxed{-\frac{7}{2}}$

7. E Long divide giving $\int_1^e \left(x - \frac{1}{x}\right)dx = \int_1^e \left(x - x^{-1}\right)dx = \left.\frac{x^2}{2} - \ln x\right]_1^e =$

$\frac{e^2}{2} - \ln e - \left(\frac{1}{2} - \ln 1\right) = \frac{e^2}{2} - 1 - \left(\frac{1}{2} - 0\right) = \boxed{\frac{e^2}{2} - \frac{3}{2}}$

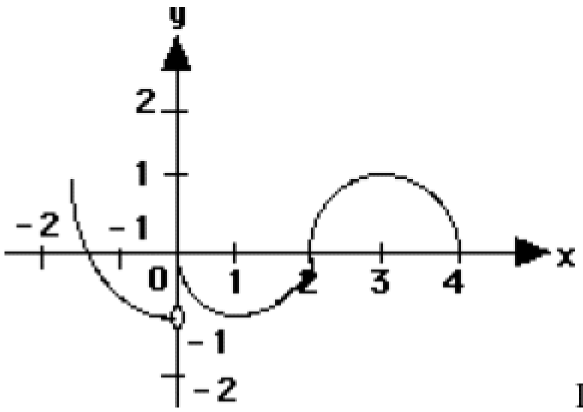
8. E Using the product rule $h'(x) = f(x)g'(x) + f'(x)g(x)$, but $h'(x) = f(x)g'(x)$ as given. Thus, $f'(x)(g(x) = 0$. Since $g(x) > 0, f'(x) = 0 \to f(x)$ is a constant. Thus, since $f(0) = 1 \to f(1) = \boxed{1}$. (Only constants have 0 derivative).

9. D Look at the graph and the area under it is about 5 of the boxes that are $6\ hours\ by\ 100\ {}^{barrels}\!/\!_{hour} = \boxed{3000}\ barrels$

10. D Instantaneous rate of change is the derivative at the point.
$f'(x) = \frac{(x-1)(2x) - (x^2-2)(1)}{(x-1)^2} = \frac{x^2 - 2x + 2}{(x-1)^2} \to f'(2) = \frac{2^2 - 2(2) + 2}{(2-1)^2} = \boxed{2}$

11. A If f(x) is linear, the $f(x) = mx + b \to f'(x) = m \to f''(x) = 0$. The integral of 0 is $\boxed{0}$.

12. E Left limit $\lim_{x\to 2-}(\ln x) = \ln 2$ Right limit $\lim_{x\to 2+}\left(x^2 \ln x\right) = 4\ln 2$ $\boxed{\ln 2 \neq 4\ln 2}$



13. B Dsicontinuity at x = 0 and vertical tangent at x = 2 means there is no slope there. $\boxed{x = \{0,2\}}$

14. C $v(t) = x'(t) = 2t - 6$ The particle is at rest if $v(t) = 0$ $2t - 6 = 0 \to \boxed{t = 3}$

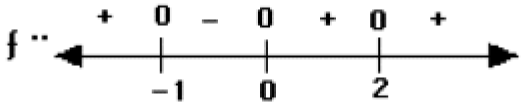
15, D Using 2nd Fundamental Theorem of Calculus
$\frac{d}{dx}\left(f(x) = \int_0^x \sqrt{t^3+1}\ dt\right) \to f'(x) = \sqrt{x^3+1} \to f'(2) = \sqrt{2^3+1} = \boxed{3}$

16. E Using the chain rule $\cos\left(e^{-x}\right)\dfrac{d}{dx}\left(e^{-x}\right)=\boxed{-\left(e^{-x}\right)\cos\left(e^{-x}\right)}$

17. D The graph shows . $f(1)=0$ Since function is increasing at $x=1, f'(x)>0$. Since function is concave down at $x=1, f''(x)<0$. In terms of the answers, this means $\boxed{f''(1)<f(1)<f'(1)}$.

18. B Derivative is $y'=1-\sin x$. So slope, m at $(0,1)$ would be $1-0=1$.
$y=mx+b \rightarrow y=1x+1 \rightarrow \boxed{y=x+1}$. Remember $(0,b)\rightarrow b$ is y-intercept.

19. C Inflection points have 2nd derivative = 0 and change of signs.
$f''(x)=0$ *at* $x=\{-1,0,2\}$ Make a 2nd derivative chart as below.

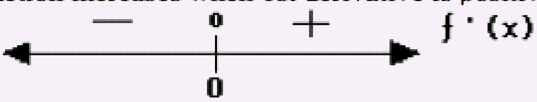


Sign changes at $\boxed{x=\{-1,0\}}$

20. A $\left.\dfrac{x^3}{3}\right]_{-3}^{k}=0\rightarrow\dfrac{k^3}{3}-\dfrac{(-3)^3}{3}=0\rightarrow k^3+27=0\rightarrow k^3=-27\rightarrow\boxed{k=-3}$

21. B $\dfrac{dy}{y}=k\ dx\rightarrow\int\dfrac{dy}{y}=\int k\ dx\rightarrow\ln y=kx+C\rightarrow e^{\ln y}=e^{kx+C}\rightarrow$
$y=e^{kx+C}\rightarrow y=e^{kx}\bullet e^{C}$ However, e^{C} is just a constant, call it
$D\rightarrow y=De^{kt}$. *Let* $D=2\rightarrow\boxed{y=2e^{kt}}$

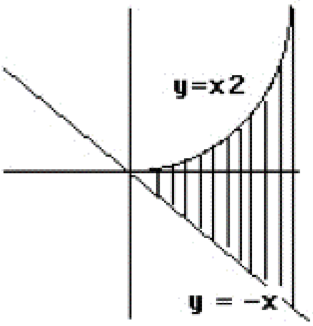
22. C $f'(x)=4x^3+2x=2x(2x^2+1)$ Set derivative to 0. $f'(x)=0$ *at* $x=0$. A function increases when 1st derivative is positive. Make 1st derivative chart.



Derivative positive $(0,\infty)$.

23. A When a function increases, its 1st derivative is positive (above the x-axis). When a function decreases, its 1st derivative is negative (below the x-axis). The derivative would be 0 at the maximum of the function. So the 1st derivative would be above the x-axis from a to the maximum point of the function, on the x-axis at the function's maximum point, then below the x-axis for the rest of the time to b. $\boxed{\text{Only graph A exhibits this.}}$

24. D $v'(t) = a(t) = 3t^2 - 6t + 12$. To find the absolute maximum of the acceleration, evaluate the value of the acceleration at the endpoints (t = 0 and t = 3) and any critical points on the interval [0,3]. The largest value for a(t) of these will be the absolute maximum of a(t) on the interval.
For the critical points of a(t) take its derivative and set it equal to 0.
$a'(t) = 6t - 6 \rightarrow 6t - 6 = 0 \rightarrow t = 1$.
Evaluate a(t) at t = 0, 1, 3. (0,12), (1,9), (3,21). The largest value is 21. $\boxed{D}$



25. D $A = \int_0^2 \left(x^2 - (x)\right)dx = \int_0^2 \left(x^2 + x\right)dx = \left.\frac{x^3}{3} + \frac{x^2}{2}\right]_0^2 =$
$\left(\frac{8}{3} + \frac{4}{2} - (0+0)\right) = \boxed{\frac{14}{3}}$

26. A By the Intermediate Value Theorem, if k were some value less than $\frac{1}{2}$, then the function would have to equal $\frac{1}{2}$ between 0 < x < 1 and again between 1 < x < 2. The only value less than $\frac{1}{2}$ is $\boxed{0}$.

27. A The average value on [0,2] of
$y = \frac{1}{2-0}\int_0^2 \left(x^2\sqrt{x^3+1}\right)dx$ Let $u = x^3 + 1$ $du = 3x^2 \rightarrow \frac{1}{2-0}\int_0^2 \left(x^2\sqrt{x^3+1}\right)dx =$
$\frac{1}{2}\left(\frac{2}{9}\left(x^3+1\right)^{3/2}\right)\Big]_0^2 = \frac{1}{9}\left[\left(9^{3/2} - 1^{3/2}\right) - (0)\right] = \frac{1}{9}(27-1) = \boxed{\frac{26}{9}}$

28. E $f'(x) = \sec^2(2x)\,(2) \rightarrow f'\left(\frac{\pi}{6}\right) = 2\sec^2\left(2\left(\frac{\pi}{6}\right)\right) = 2\sec^2\left(\left(\frac{\pi}{3}\right)\right) = 2(2)^2 = \boxed{8}$

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