Question 4

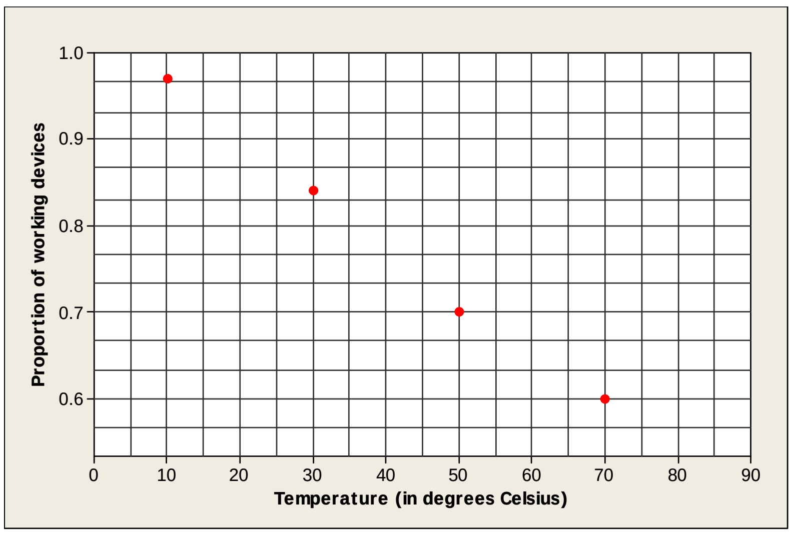
Intent of Question

The primary goals for this question were to assess a student's ability to (1) create and interpret a scatterplot and (2) estimate a proportion and the associated standard error.

Solution

Part (a):

Create a new variable p that indicates the proportion (number working / number tested) of working devices. A scatterplot of the proportion of working devices and temperature in degrees Celsius is shown below.



Part (b):

The scatterplot clearly shows a very strong, negative, linear association between the temperature and proportion of working devices at the end of the 5,000 hours. As the temperature increases, the proportion of working devices decreases.

Part (c):

The estimated proportion is the linear combination $\hat{p}_{40^\circ} = \frac{1}{2}\left(\frac{X_{30^\circ}}{n_{30^\circ}}\right) + \frac{1}{2}\left(\frac{X_{50^\circ}}{n_{50^\circ}}\right) = \frac{1}{2}\left(\frac{42}{50}\right) + \frac{1}{2}\left(\frac{21}{30}\right) = 0.77.$

Because the results for the two devices are independent, the variance of the estimated proportion

is $\mathrm{Var}(\hat{p}_{40^\circ}) = \left(\frac{1}{2}\right)^2\left(\frac{\left(\frac{42}{50}\right)\left(\frac{8}{50}\right)}{50}\right) + \left(\frac{1}{2}\right)^2\left(\frac{\left(\frac{21}{30}\right)\left(\frac{9}{30}\right)}{30}\right).$

Thus, the standard error is given by $\sqrt{\mathrm{Var}(\hat{p}_{40^\circ})} = \left(\frac{1}{2}\right)\sqrt{0.0027 + 0.007} = 0.0492.$

Question 4 (continued)

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if a scatterplot of the proportions is provided with appropriate scales and labels.

Partially correct (P) if a scatterplot of the proportions is provided with inadequate scales or inadequate labeling.

Incorrect (I) if an incorrect plot (e.g., a plot of the original counts or a nonscatterplot) is provided.

Notes:
- Students must give labels and at least two values on each scale to get full credit. One exception is that if the correct unit (°C, or similar) is given with values on the temperature axis, then this will suffice as a temperature label. (Note that the ° symbol alone does not suffice as the temperature label.)
- Labels that use "probability" instead of "proportion" or "percentage" are inadequate labels. Labels that use "estimated probability" or "sampled probability" are acceptable.

Part (b) is scored as follows:

Essentially correct (E) if the strength, direction, and type of relationship between these variables are correctly described in context.

Partially correct (P) if only two or three of the four (strength, direction, type, or context) are correctly described.

Incorrect (I) if none or one of the four (strength, direction, type, or context) is correctly described.

Note: Correlation is considered to be synonymous with linear association. Also, a student could give a correct interpretation of an incorrect plot (e.g., a plot of counts).

Part (c) is scored as follows:

Essentially correct (E) if an appropriate estimate and its standard error are calculated correctly with work shown.

Partially correct (P) if one of the two calculations is correct with work shown.

Incorrect (I) if correct answers are provided with no work shown *OR* correct answers are provided using inappropriate methods.

Question 4 (continued)

Notes:

- Students can give two possible approaches to "averaging the estimates at 30°C and 50°C." The first is the arithmetic mean $\hat{p}_{40^{\circ}} = \dfrac{\frac{42}{50} + \frac{21}{30}}{2} = 0.77$ with its standard error as given in the model solution. The second is the pooled estimate $\hat{p}_{40^{\circ}} = \dfrac{42 + 21}{50 + 30} = 0.7875.$ The standard error of this pooled estimate is calculated as follows:

$$\text{Var}\left(\frac{X_{30^{\circ}} + X_{50^{\circ}}}{n_{30^{\circ}} + n_{50^{\circ}}}\right) = \frac{1}{80^2}\left(n_{30^{\circ}}\, p_{30^{\circ}} q_{30^{\circ}} + n_{50^{\circ}}\, p_{50^{\circ}} q_{50^{\circ}}\right) = \frac{1}{80^2}\left(50\left(\frac{42}{50}\right)\left(\frac{8}{50}\right) + 30\left(\frac{21}{30}\right)\left(\frac{9}{30}\right)\right) = 0.00203.$$

$$\text{SE} = \sqrt{\text{Var}\left(\frac{X_{30^{\circ}} + X_{50^{\circ}}}{n_{30^{\circ}} + n_{50^{\circ}}}\right)} = \sqrt{0.00203} = 0.0451.$$

- Any standard error of a single proportion is not acceptable.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and no parts partially correct

OR

No parts essentially correct and two parts partially correct

4A1

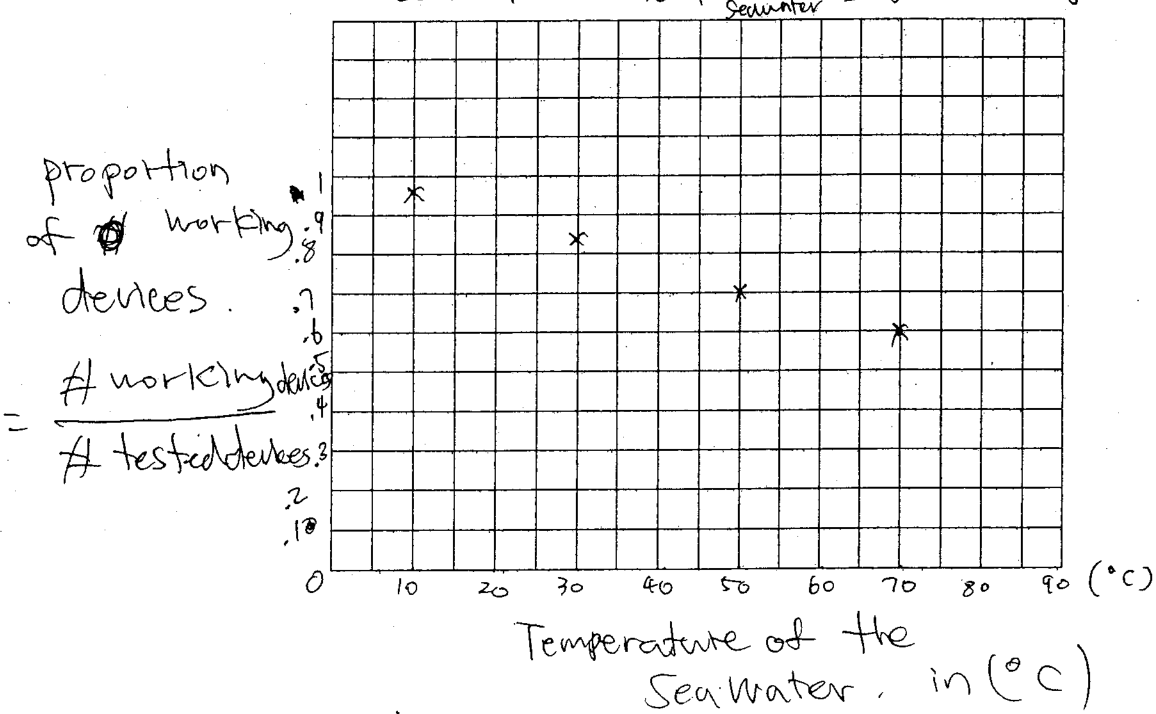
4. An experiment was conducted to study the effect of temperature on the reliability of an electronic device used in an undersea communications system. The experiment was done in a laboratory where tanks of seawater were maintained at either 10°C, 30°C, 50°C, or 70°C. After the electronic devices were submerged in the tanks for 5,000 hours, each device was inspected to determine if it was still working. The following table provides information on the number of devices tested at each temperature and the number of working devices at the end of the 5,000-hour test.

Seawater temperature	10°C	30°C	50°C	70°C
Number of working devices	29	42	21	12
Number of devices tested	30	50	30	20
	.9666	.84	.7	.6

You may assume that the result for any single device is not influenced by the result for any other device.

(a) Using the information in the table, construct a scatterplot that would be useful for showing the effect of water temperature on the ability of the devices to work for at least 5,000 hours. electronic

scatterplot of temperature (seawater) vs # of working devices.



x represents the points.

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4A2

(b) Comment on any trend or pattern that is revealed by the scatterplot you constructed.

Even though there were only 4 different conditions, (temperature of ~~20, 30~~ 10, 30, 50, 70 °Cs) The scatter plot shows strong negative correlation between temp and proportion of working electronic devices.

The correlation suggest that there are strong negative linear association between temp, and p of working devices.

(c) An estimate of the proportion of devices that would work after 5,000 hours of submersion in 40°C seawater can be obtained by averaging the estimates at 30°C and 50°C. Compute this estimate and the associated standard error.

at 30°C – $p = .84$

at 50°C – $p = .7$

at 40°C – $\frac{.84 + .7}{2} = .77$

Standar Error of this will be

$$Std = \frac{\sigma_1 + \sigma_2}{2} = \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} / 2$$

$$SE = \sqrt{\frac{\hat{P}_{30°C}(1 - \hat{P}_{30°C})}{50} + \frac{\hat{P}_{50°C}(1 - \hat{P}_{50°C})}{30}} \Big/ 2 = .04921$$

X ^s No hats.

for $\hat{P}_{30°C}$.
$n = 50$
$p = .84$

for $\hat{P}_{50°C}$
$n = 30$
$p = .7$

GO ON TO THE NEXT PAGE.

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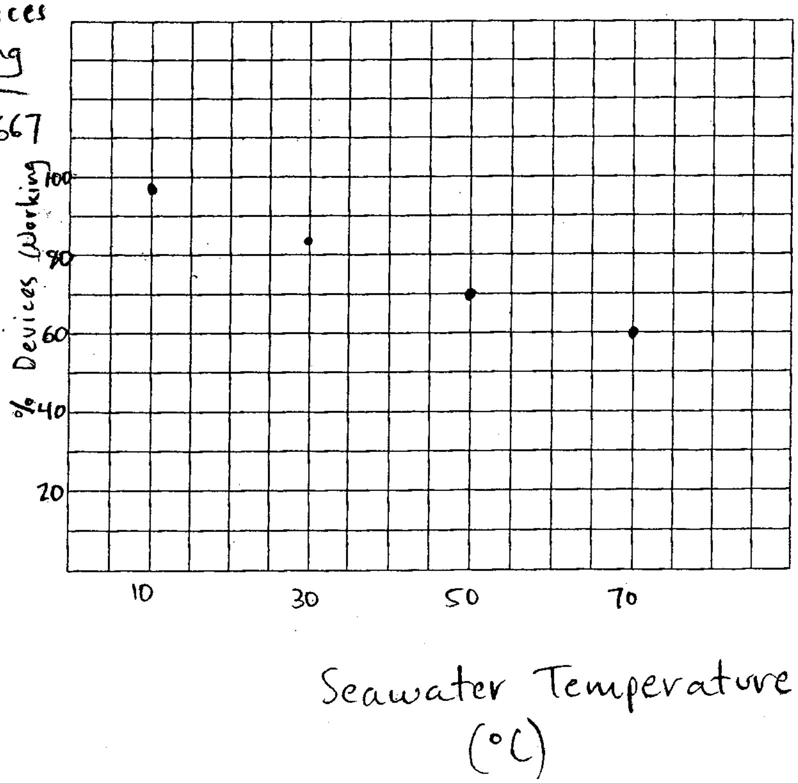
You may assume that the result for any single device is not influenced by the result for any other device.

(a) Using the information in the table, construct a scatterplot that would be useful for showing the effect of water temperature on the ability of the devices to work for at least 5,000 hours.

Temp (°C)	% of Devices Working
10	$\frac{29}{30} = 0.9667$
30	$\frac{42}{50} = 84$
50	$\frac{21}{30} = 70$
70	$\frac{12}{20} = 60$



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(b) Comment on any trend or pattern that is revealed by the scatterplot you constructed.

As the seawater temperature increases, the percent of devices tested that work steadily decreases. This implies that there may be an association between seawater temperature and electronic device reliability

(c) An estimate of the proportion of devices that would work after 5,000 hours of submersion in 40°C seawater can be obtained by averaging the estimates at 30°C and 50°C. Compute this estimate and the associated standard error.

% working at 30° = 84%

% working at 50° = 70%

% working at 40% = $\frac{84+70}{2}$

= 77%

The estimate of the proportion of devices working at 40°C is 77%.

$SE(\text{at } 30°) = \sqrt{\frac{0.84(0.16)}{50}} = S_{30}$ $SE(\text{at } 50°) = \sqrt{\frac{0.7(0.3)}{30}} = S_{50}$

$SE(\text{at } 40°) = S_{40} = \frac{\sqrt{S_{30}^2 + S_{50}^2}}{2}$

$\boxed{S_{40} = 0.049}$

The standard error at 40°C is 0.049.

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4C1

4. An experiment was conducted to study the effect of temperature on the reliability of an electronic device used in an undersea communications system. The experiment was done in a laboratory where tanks of seawater were maintained at either 10°C, 30°C, 50°C, or 70°C. After the electronic devices were submerged in the tanks for 5,000 hours, each device was inspected to determine if it was still working. The following table provides information on the number of devices tested at each temperature and the number of working devices at the end of the 5,000-hour test.

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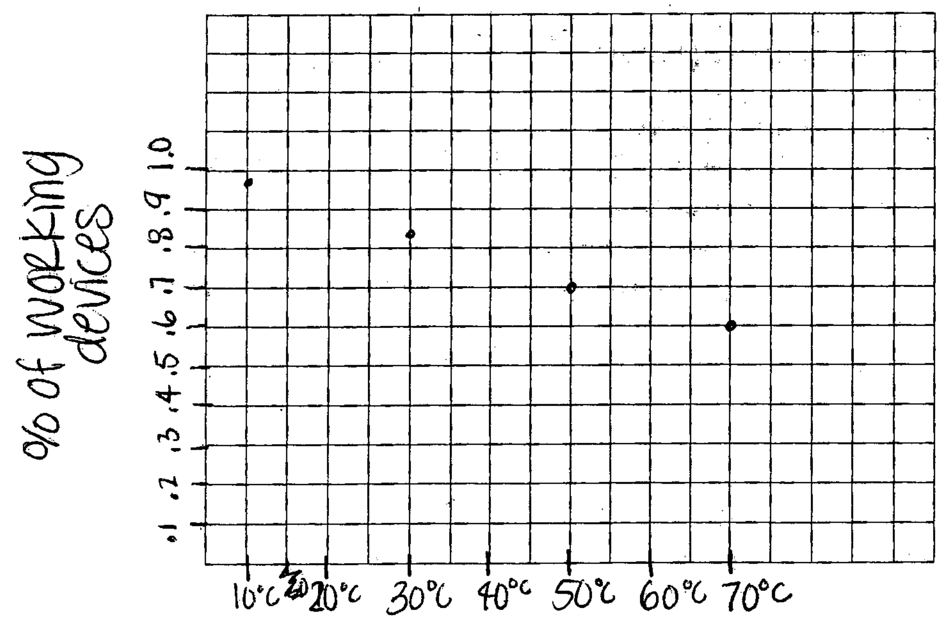
You may assume that the result for any single device is not influenced by the result for any other device.

(a) Using the information in the table, construct a scatterplot that would be useful for showing the effect of water temperature on the ability of the devices to work for at least 5,000 hours.



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(b) Comment on any trend or pattern that is revealed by the scatterplot you constructed.

It appears to be a very linear pattern of the scatter plot. Which means as the °C goes up the ability of the device to work for at least 5,000 hours decreases at a constant rate.

(c) An estimate of the proportion of devices that would work after 5,000 hours of submersion in 40°C seawater can be obtained by averaging the estimates at 30°C and 50°C. Compute this estimate and the associated standard error.

$42 + 21 = \frac{63}{2} = 31.5$ working

$50 + 30 = \frac{80}{2} = 40$ tested $\rightarrow \frac{31.5}{40} =$ .7875 = the prop of devices that would work after 5,000 hours @ 40°C seawater

$$\sigma_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{.7875(.2125)}{40}} = .0647$$

$\sigma_{\hat{p}} = .0647 =$ std @ 40°C

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Question 4

Overview

The primary goals for this question were to assess a student's ability to (1) create and interpret a scatterplot and (2) estimate a proportion and the associated standard error.

Sample: 4A
Score: 4

Each part of this response is complete, concise, and clearly communicated. In part (a) the student constructs a scatterplot, with a title, using the "proportion of working devices" on the *y*-axis, properly labeled, and the seawater temperature on the *x*-axis. The student indicates the proportion of working devices for each temperature in the last row of the printed data table; part (a) was thus scored as essentially correct. In part (b) the student correctly indicates that the resulting scatterplot from part (a) shows a "strong negative correlation" between the water temperature and the proportion of working devices. The student continues with a similar statement, indicating a "linear association." This was scored as an essentially correct response to part (b). In part (c) the student gives a correct estimate of the proportion of devices that would work after 5,000 hours of submersion in 40°C seawater and the correct standard error of that estimate. This was scored an essentially correct response. The entire answer, inclusive of all three parts, was considered a complete response and was awarded 4 points.

Sample: 4B
Score: 3

Two of the three parts are complete and clearly communicated. In part (a) the student calculates the proportion of working devices at each of the four temperatures and uses these values to construct the correct scatterplot. The student labels the *y*-axis as the "% of Devices Working" and the *x*-axis as "Seawater Temperature (°C)." This was scored as an essentially correct response. In part (b) the student clearly indicates, in context, the direction of the relationship between the seawater temperature and the proportion of working devices. The phrase "steadily decreases" suffices for the linear association component. The student fails to indicate the strength of the relationship. This covers three of the four necessary components for part (b); hence it was scored as a partially correct response. In part (c) the student arrives at a correct estimate of the proportion of devices that would work after 5,000 hours of submersion in 40°C seawater and the correct standard error of that estimate. This was scored as an essentially correct response. The answer as a whole was judged a substantial response and received 3 points.

Sample: 4C
Score: 2

One of the three parts of this response was scored as essentially correct. In part (a) the student correctly constructs a scatterplot of the proportion of working devices after 5,000 hours, placing the proportion of devices working on the *y*-axis and the temperature on the *x*-axis. Note that the student does not have to give any additional labeling on the *x*-axis, as the scale provides values that are in the appropriate units (°C). This was scored as an essentially correct response. In part (b) the student describes the "linear pattern" and the direction of the relationship in context but neglects to assess the strength of the relationship. This covers three of the four necessary components of part (b) and was scored as a partially correct response. In part (c) the student gives the pooled estimate of the proportion of working devices at 40°C but fails to determine a correct standard error. This was scored as a partially correct response to part (c). Overall, this answer was deemed a developing response and was awarded 2 points.