2017

AP CollegeBoard

AP Calculus AB

Sample Student Responses and Scoring Commentary

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AP® CALCULUS AB
2017 SCORING GUIDELINES

Question 2

(a) $\int_0^2 f(t)\,dt = 20.051175$

20.051 pounds of bananas are removed from the display table during the first 2 hours the store is open.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $f'(7) = -8.120$ (or -8.119)

After the store has been open 7 hours, the rate at which bananas are being removed from the display table is decreasing by 8.120 (or 8.119) pounds per hour per hour.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{meaning} \end{cases}$

(c) $g(5) - f(5) = -2.263103 < 0$

Because $g(5) - f(5) < 0$, the number of pounds of bananas on the display table is decreasing at time $t = 5$.

$2 : \begin{cases} 1 : \text{considers } f(5) \text{ and } g(5) \\ 1 : \text{answer with reason} \end{cases}$

(d) $50 + \int_3^8 g(t)\,dt - \int_0^8 f(t)\,dt = 23.347396$

23.347 pounds of bananas are on the display table at time $t = 8$.

$3 : \begin{cases} 2 : \text{integrals} \\ 1 : \text{answer} \end{cases}$

2A, 2 2 2 2 2 2 2 2 2 2 2A

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \le 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln\left(t^2 + 2t\right) \text{ for } 3 < t \le 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

(a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?

$\int_0^2 f(t)dt = 20.051$ lbs.

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(b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.

$f'(7) = -8.119$ lbs. per hour^2

At time $t = 7$, the rate at which customers remove bananas from the display table is decreasing at a rate of 8.119 lbs. per hour per hour.

Continue question 2 on page 7.

(c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.

$f(5) = 13.796$
$g(5) = 11.532$

The number of pounds of bananas on the display table is decreasing at time $t=5$, because $f(5) > g(5)$.

(d) How many pounds of bananas are on the display table at time $t = 8$?

$$50 - \int_0^8 f(t)\,dt + \int_3^8 g(t)\,dt = 23.347 \text{ lbs.}$$

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2B, 2 2 2 2 2 2 2 2 2 2 2B,

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

rate at which bananas are removed

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \le 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln\left(t^2 + 2t\right) \text{ for } 3 < t \le 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

(a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?

$$\int_0^2 \left(10 + (0.8t)\sin\left(\frac{t^3}{100}\right)\right)dt = 30.3867 \text{ bananas}$$

(b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.

$$f(t) = 10 + 0.8t\cdot\sin\left(\frac{t^3}{100}\right)$$

$$f'(t) = 0 + \left[(0.8)\sin\left(\frac{t^3}{100}\right) + \left(\cos\frac{t^3}{100}\right)\left(\frac{3t^2\cdot 100}{(100)^2}\right)(0.8t)\right]$$

$$f'(t) = 0.8\sin\left(\frac{t^3}{100}\right) + \frac{300t^2}{100^2}\left[\cos\left(\frac{t^3}{100}\right)\right](0.8t)$$

$$f'(7) = -8.11954$$

The rate at which the banana pile is decreasing

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Continue question 2 on page 7.

(c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.

$f(5) = 12.38$ pounds / hr

$g(5) = 11.53$ pounds / hr

The number of pounds of bananas is decreasing at $t = 5$ because the rate at which they are being removed $[f(t)]$ is greater than the rate at which it is being added $[g(t)]$.

(d) How many pounds of bananas are on the display table at time $t = 8$?

$$50 - \int_0^8 \left[10 + (0.8t)\sin\left(\frac{t^3}{100}\right)\right]dt + \int_3^8 \left[3 + 2.4\ln(t^2 + 2t)\right]dt$$

$50 - 85.6167 + 58.9641 =$ 23.3474 pounds of bananas

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2q 2 2 2 2 2 2 2 2 2 2 2c

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \le 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln\left(t^2 + 2t\right) \text{ for } 3 < t \le 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

(a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?

$= 50 - \int_0^2 f(t)\,dt$

29.9 pounds

approximately 30 pounds of bananas are removed from the display table during the first 2 hours

(b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.

$\frac{d}{dx}(f(x))\Big|_{x=7}$

≈ -8.12 pounds per hour squared

$f'(t)$ is the acceleration rate at which bananas are removed from the display table measured in pounds per hour squared

$-f'(t)$ is the derivative of $f(t)$

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Continue question 2 on page 7.

2C2 2 2 2 2 2 2 2 2 2 2 2C2

(c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.

$F'(t) > 0$ therefore the number of pounds of bananas on the display is increasing at $t = 5$

$\frac{d}{dt}(F(t))\Big|_{x=5}$

$\approx 1.7 > 0$

(d) How many pounds of bananas are on the display table at time $t = 8$?

$\int_0^8 (F(t))\,dt$

≈ 85.6

approximately, there are 86 pounds of bananas on the display table at $t = 8$

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AP® CALCULUS AB
2017 SCORING COMMENTARY

Question 2

Overview

The context for this problem is the removal and restocking of bananas on a display table in a grocery store during a 12-hour period. Initially, there are 50 pounds of bananas on the display table. The rate at which customers remove bananas from the table is modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \quad \text{for } 0 < t \le 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. Three hours after the store opens, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln\left(t^2 + 2t\right) \text{ for } 3 < t \le 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened. In part (a) students were asked how many pounds of bananas are removed from the display table during the first 2 hours the store is open. Students needed to realize that the amount of bananas removed from the table during a time interval is found by integrating the rate at which bananas are removed across the time interval. Thus, students needed to express this amount as $\int_0^2 f(t)\,dt$ and use the graphing calculator to produce a numeric value for this integral. [LO 3.4E/EK 3.4E1] In part (b) students were asked to find $f'(7)$ and, using correct units, explain the meaning of $f'(7)$ in the context of the problem. Students were expected to use the graphing calculator to evaluate the derivative, and explain that the rate at which bananas are being removed from the display table 7 hours after the store has been open is decreasing by 8.120 pounds per hour per hour. [LO 2.3A/EK 2.3A1, LO 2.3D/EK 2.3D1] In part (c) students were asked to determine, with reason, whether the number of pounds of bananas on the display table is increasing or decreasing at time $t = 5$. This can be determined from the sign of the difference between the rate at which bananas are added to the table and the rate at which they are removed from the table. Thus, students needed to evaluate the difference $g(5) - f(5)$ on the graphing calculator and report that the number of pounds of bananas on the display table is decreasing because this value is negative. [LO 2.2A/EK 2.2A1] In part (d) students were asked how many pounds of bananas are on the display table at time $t = 8$. The number of pounds of bananas added to the table by time $t = 8$ is given by $\int_3^8 g(t)\,dt$, and the number of pounds of bananas removed from the table by that time is given by $\int_0^8 f(t)\,dt$. Thus, using that there were initially 50 pounds of bananas on the table, the expression $50 + \int_3^8 g(t)\,dt - \int_0^8 f(t)\,dt$ gives the number of pounds of bananas on the table at time $t = 8$. Students needed to evaluate this expression using the numeric integration capability of the graphing calculator. [LO 3.4E/EK 3.4E1] This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, building notational fluency, and communicating.

Question 2 (continued)

Sample: 2A
Score: 9

The response earned all 9 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and 3 points in part (d). In part (a) the student earned the first point for the definite integral $\int_0^2 f(t)\,dt$ and the second point for computing the correct value. In part (b) the student earned the first point for correctly computing $f'(7) = -8.119$. The student earned the second point by having the appropriate units, mentioning the time $t = 7$, and correctly describing the meaning in context as "the rate at which customers remove bananas from the display table." In part (c) the student earned the first point for the expressions $f(5)$ and $g(5)$ on the left-hand side of the equations in the first two lines. The student earned the second point for reaching the correct conclusion of "decreasing" by comparing these two correct values. In part (d) the student earned both integral points: 1 point for each of the definite integrals $\int_0^8 f(t)\,dt$ and $\int_3^8 g(t)\,dt$. The student earned the third point for computing the correct value.

Sample: 2B
Score: 6

The response earned 6 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and 3 points in part (d). In part (a) the student earned the first point for the definite integral $\int_0^2 \left(10 + (0.8t)\sin\left(\frac{t^3}{100}\right)\right) dt$ on the left-hand side of the equation. The student did not earn the second point because the value produced is incorrect. In part (b) the student earned the first point in the fourth line for $f'(7) = -8.11954$. The student did not earn the second point because units are not included, and no mention is made of the time $t = 7$ in the explanation. In part (c) the student earned the first point for the expressions $f(5)$ and $g(5)$ on the left-hand side of the equations in the first two lines. The student reports each of these values to only two decimal places, which is acceptable because this is intermediate work. However, the value for $f(5)$ is incorrect, so the student is not eligible for the second point. In part (d) the student earned both integral points: 1 point for each of the definite integrals in the first line. The student earned the third point for the expression $50 - 85.6167 + 58.9641$. The student chooses to simplify and does so correctly because the boxed answer is accurate to at least three decimal places.

Sample: 2C
Score: 3

The response earned 3 points: 1 point in part (a), 1 point in part (b), no points in part (c), and 1 point in part (d). In part (a) the student earned the first point for the definite integral in the first line. The student did not earn the second point because the value produced is incorrect. In part (b), although the student has changed the variable to *x*, the student is not penalized for this and earned the first point in the second line with the value -8.12. This is accurate to three decimal places because the thousandths digit is 0. The student did not earn the second point because the student does not mention the time $t = 7$ in the explanation, and the generic term "acceleration" does not capture the context of the problem. In part (c) the student did not earn the first point because only $f'(5)$ is considered. The student is not eligible for the second point. In part (d) the student earned 1 of the 2 integrals points for the definite integral in the first line. The student does not have a definite integral involving $g(t)$, so the student is not eligible for the third point.