2017

AP® CollegeBoard

AP Calculus AB

Sample Student Responses and Scoring Commentary

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- ☑ Scoring Guideline
- ☑ Student Samples
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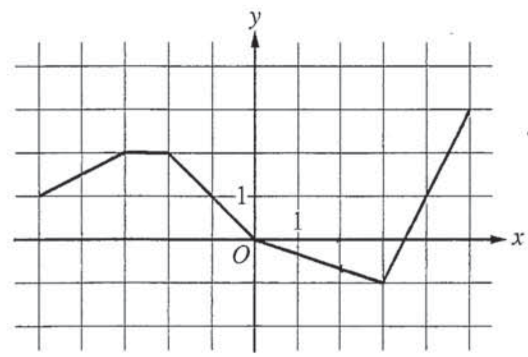
Question 6

(a) $f'(x) = -2\sin(2x) + \cos x\, e^{\sin x}$ $f'(\pi) = -2\sin(2\pi) + \cos\pi\, e^{\sin\pi} = -1$	$2 : f'(\pi)$
(b) $k'(x) = h'(f(x)) \cdot f'(x)$ $k'(\pi) = h'(f(\pi)) \cdot f'(\pi) = h'(2) \cdot (-1)$ $= \left(-\frac{1}{3}\right)(-1) = \frac{1}{3}$	$2 : \begin{cases} 1 : k'(x) \\ 1 : k'(\pi) \end{cases}$
(c) $m'(x) = -2g'(-2x) \cdot h(x) + g(-2x) \cdot h'(x)$ $m'(2) = -2g'(-4) \cdot h(2) + g(-4) \cdot h'(2)$ $= -2(-1)\left(-\frac{2}{3}\right) + 5\left(-\frac{1}{3}\right) = -3$	$3 : \begin{cases} 2 : m'(x) \\ 1 : m'(2) \end{cases}$
(d) g is differentiable. $\Rightarrow$ g is continuous on the interval $[-5, -3]$. $\frac{g(-3) - g(-5)}{-3 - (-5)} = \frac{2 - 10}{2} = -4$ Therefore, by the Mean Value Theorem, there is at least one value c, $-5 < c < -3$, such that $g'(c) = -4$.	$2 : \begin{cases} 1 : \frac{g(-3) - g(-5)}{-3 - (-5)} \\ 1 : \text{justification, using Mean Value Theorem} \end{cases}$

6A₁ 6A₁

NO CALCULATOR ALLOWED

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

$m = -\frac{1}{3}x$

$-2/3$

$\frac{-1-0}{3-0} = -\frac{1}{3}$

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

$f'(x) = -2\sin(2x) + e^{\sin x}(\cos x)$

$f'(\pi) = -2\sin(2\pi) + e^{\sin \pi}(\cos \pi)$

$= 0 + 1(-1)$

$= \underline{-1}$

(-1, 0) (1, 0)

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

$\cos(2\pi) + e^{\sin \pi}$

$1 + 1$

$k'(x) = h'(f(x))(f'(x))$

$k'(\pi) = h'(f(\pi))\, f'(\pi)$

$= h'(2)(-1)$

$= 1/3$

Continue question 6 on page 21.

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

$$m'(x) = [g'(-2x)(-2) \cdot h(x)] + [h'(x) \cdot g(-2x)]$$
$$m'(2) = [2g'(-4) \cdot h(2)] + [h'(2) \cdot g(-4)]$$
$$= (2 \cdot -2/3) + (-1/3 \cdot 5)$$
$$-\frac{4}{3} + -\frac{5}{3}$$
$$= -\frac{9}{3}$$
$$= \underline{-3}$$

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

$g(x)$ = differentiable $\rightarrow$ $g(x)$ continuous $\therefore$
Mean value theorem states that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\frac{g(-3) - g(-5)}{-3 + 5} = \frac{2 - 10}{2}$$
$$= \frac{-8}{2}$$
$$= -4$$

yes, there's a number c in the closed interval such that $g'(c) = -4$.

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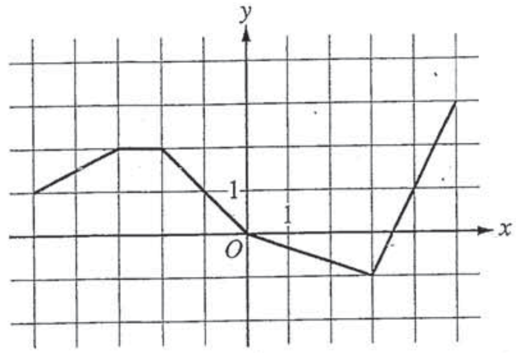
6B₁ 6B₁

NO CALCULATOR ALLOWED

x	$g(x)$	$g'(x)$
−5	10	−3
−4	5	−1
−3	2	4
−2	3	1
−1	1	−2
0	0	−3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

$f(x) = \cos(2x) + e^{\sin x}$

$f'(x) = -2\sin(2x) + (\cos x)e^{\sin x}$

$f'(\pi) = -2\sin(2\pi) + (\cos\pi)e^{\sin\pi}$

$f'(\pi) = -2(0) + (-1)e^{(0)}$

$f'(\pi) = 0 + -1(1) = -1$

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

$k(x) = h(f(x))$

$k'(x) = h'(f(x))\, f'(x)$

$k'(\pi) = h'(f(\pi))\, f'(x)$

$k'(\pi) = [h'(2)](-1)$

$k'(\pi) = (-\frac{1}{3})(-1)$

$f(\pi) = \cos(2\pi) + e^{\sin\pi}$

$f(\pi) = 1 + e^0 = 1+1 = 2$

Continue question 6 on page 21.

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

$m(x) = g(-2x) \cdot h(x)$

$m'(x) = -2g'(-2x) \cdot h(x) + h'(x) \cdot g(-2x)$

$m'(2) = -2g'(-4) \cdot h(2) + h'(2) \cdot g(-4)$

$m'(2) = -2[-1] \cdot \frac{2}{3} + \left(\frac{1}{3}\right) \cdot 5$

$m'(2) = \frac{8}{3} + \frac{5}{3} = \frac{13}{3}$

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

No, there isn't a number c in the interval [-5,-3] such that g'(c)=4 because the values of g'(x) are increasing from [-5,-3] and show no sign that the values will decreasing. This can be justified by the midpoint theorem as

x	$g'(x)$
-5	-3
-4	1
-3	4
c	$-3 \leq x \leq 4$

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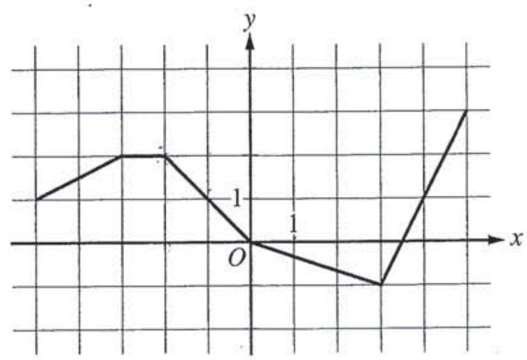
6C1 6C1

NO CALCULATOR ALLOWED

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

$$f'(x) = -2\sin(2x) + e^{\sin x}(\cos(x))$$

$$f'(\pi) = -2\sin(2\pi) + e^{\sin\pi}\cos(\pi)$$

$$0 \quad + 1(-1)$$

$$f'(\pi) = \boxed{-1}$$

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

$$k'(x) = h'(f(x))f'(x)$$

$$k'(\pi) = h'(f(\pi))f'(\pi)$$

Continue question 6 on page 21.

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6C2 6C2

NO CALCULATOR ALLOWED

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

$m'(x) = g'(-2x)(-2) \cdot h'(x)$

$m'(2) = g'(-4) -2 \cdot h'(-2)$

$m'(2) = (4 -2) \cdot -1$

$m'(2) = 2 \cdot -1$

$\boxed{m'(2) = -2}$

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

No, because there are no points in that interval with a slope of -4.

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AP® CALCULUS AB
2017 SCORING COMMENTARY

Question 6

Overview

This problem deals with multiple functions. Function f is defined by $f(x) = \cos(2x) + e^{\sin x}$. Function g is differentiable and values of $g(x)$ and $g'(x)$ corresponding to integer values of x from $x = -5$ to $x = 0$, inclusive, are given in a table. Function h is defined on $[-5, 5]$ and the graph of h, comprised of five line segments, is given. In part (a) students were asked for the slope of the line tangent to the graph of f at $x = \pi$. Using the sum and chain rules for differentiation and the derivatives of trigonometric and exponential functions to differentiate $f(x)$, students needed to evaluate $f'(\pi)$ to find the slope of the tangent line. [LO 2.1C/EK 2.1C2-2.1C4, LO 2.3B/EK 2.3B1] In part (b) the function k is defined by $k(x) = h(f(x))$, and students were asked to find $k'(\pi)$. Students needed to apply the chain rule and determine the value of $h'(2)$ from the graph of h to arrive at the value for $k'(\pi)$. [LO 2.1C/EK 2.1C4, LO 2.2A/EK 2.2A2] In part (c) the function m is defined by $m(x) = g(-2x) \cdot h(x)$, and students were asked to find $m'(2)$. Students needed to apply the product and chain rules for differentiation, find values for $g(-4)$ and $g'(-4)$ in the table for g, and use the graph of h to determine $h(2)$ and $h'(2)$, to find $m'(2) = -2g'(-4) \cdot h(2) + g(-4) \cdot h'(2) = -3$. [LO 2.1C/EK 2.1C3-2.1C4, LO 2.2A/EK 2.2A2, LO 2.3B/EK 2.3B1] In part (d) students were asked to determine whether there is a number c in the interval $[-5, -3]$ such that $g'(c) = -4$, and to justify their answers. Using the table for g, students should have confirmed that $\frac{g(-3) - g(-5)}{-3 - (-5)} = -4$. Given that g is differentiable, students should have concluded that g is continuous on $[-5, -3]$ and, thus, recognize that the hypotheses for the Mean Value Theorem are satisfied, and answered in the affirmative that a number c exists in the interval $[-5, -3]$ such that $g'(c) = -4$. [LO 1.2B/EK 1.2B1, LO 2.4A/EK 2.4A1] This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating.

Sample: 6A
Score: 9

The response earned all 9 points: 2 points in part (a), 2 points in part (b), 3 points in part (c), and 2 points in part (d). In part (a) the student presents a correct expression for $f'(x)$ in line 1 and earned the points for $f'(\pi)$ in line 4. The expression in line 2 would have also earned the points for $f'(\pi)$ without simplification. The student chooses to simplify and does so correctly. Both points were earned. In part (b) the student earned the point for $k'(x)$ in line 1 and earned the point for $k'(\pi)$ in line 4. In part (c) the student earned both points for $m'(x)$ in line 1. The student would have earned the point for $m'(2)$ in line 3. The student chooses to simplify and does so correctly, so the student earned the point. In part (d) the student earned the point for the difference quotient with the statement $\frac{g(-3) - g(-5)}{-3 + 5}$. The student confirms the conditions for the Mean Value Theorem in the first line, goes on to connect the difference quotient with the value -4, and draws the appropriate conclusion for the Mean Value Theorem. The student earned the justification point.

Question 6 (continued)

Sample: 6B
Score: 6

The response earned 6 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and no points in part (d). In part (a) the student presents a correct expression for $f'(x)$ in line 2 and earned the points for $f'(\pi)$ in line 5. The expression in line 3 would have also earned the points for $f'(\pi)$ without simplification. The student chooses to simplify and does so correctly. Both points were earned. In part (b) the student earned the point for $k'(x)$ in line 2 on the left and earned the point for $k'(\pi)$ in the last line on the left, after correctly evaluating and using $h'(2)$. In part (c) the student earned both points for $m'(x)$ in line 2. The student is at first eligible to earn the point for $m'(2)$ in line 4, but the student makes errors in evaluating $h(2)$ and $h'(2)$. The student did not earn the third point. In part (d) the student does not present a difference quotient and never engages with the Mean Value Theorem. As a result, the student did not earn any points.

Sample: 6C
Score: 3

The response earned 3 points: 2 points in part (a), 1 point in part (b), no points in part (c), and no points in part (d). In part (a) the student presents a correct expression for $f'(x)$ in line 1 and earned the points for $f'(\pi)$ in line 4. The expression in line 2 would have also earned the points for $f'(\pi)$ without simplification. The student chooses to simplify and does so correctly. Both points were earned. In part (b) the student earned the point for $k'(x)$ in line 1. The student substitutes π for x in line 2 but does not evaluate the expression. The student did not earn the point for $k'(\pi)$. In part (c) the student does not present an expression that uses the product rule. Thus, the student is not eligible to earn any points. In part (d) the student does not present a difference quotient, so that point was not earned. Because the student's conclusion is incorrect, the student is not eligible for the justification point.