2017

AP CollegeBoard

AP Statistics

Sample Student Responses and Scoring Commentary

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AP® STATISTICS
2017 SCORING GUIDELINES

Question 5

Intent of Question

The primary goal of this question was to assess a student's ability to identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question. More specific goals were to assess a student's ability to (1) state appropriate hypotheses; (2) identify the appropriate statistical test procedure and check appropriate conditions for inference; (3) calculate the appropriate test statistic and *p*-value; and (4) draw an appropriate conclusion, with justification, in the context of the study.

Solution

Step 1: State a correct pair of hypotheses.

The null hypothesis is that age group at diagnosis and gender are independent (that is, they are not associated) for the population of people currently being treated for schizophrenia.

The alternative hypothesis is that age group at diagnosis and gender are not independent for the population of people currently being treated for schizophrenia.

Step 2: Identify a correct test procedure (by name or formula) and check appropriate conditions.

The appropriate test is a chi-square test of independence.

The conditions for this test are satisfied because:

1. The question states that the sample was randomly selected.
2. The expected counts for the eight cells of the table are at least 5, as seen in the following table, with expected counts shown below observed counts.

	Age at Diagnosis				
	20 to 29	30 to 39	40 to 49	50 to 59	Total
Women	46	40	21	12	119
	56.91	36.22	17.25	8.62	
Men	53	23	9	3	88
	42.09	26.78	12.75	6.38	

Step 3: Find the value of the test statistic and the *p*-value.

The test statistic is calculated as $\chi^2 = \sum \frac{(O-E)^2}{E}$, or

$$\chi^2 = 2.093 + 0.395 + 0.817 + 1.322 + 2.830 + 0.534 + 1.105 + 1.788 = 10.884.$$

The *p*-value is $P(\chi^2 \geq 10.884) = 0.012$, based on $(4-1) \times (2-1) = 3$ degrees of freedom.

Question 5 (continued)

Step 4: State the conclusion in context, with linkage to the *p*-value.

Because the *p*-value is very small (for instance much smaller than $\alpha = 0.05$), we reject the null hypothesis and conclude that the sample data provide strong evidence that there is an association between age group at diagnosis and gender for the population currently being treated for schizophrenia.

Scoring

This question is scored in three sections. Section 1 consists of steps 1 and 2 (stating the correct hypotheses, identifying the appropriate test procedure, and checking the technical conditions); section 2 consists of step 3 (performing the correct mechanics); and section 3 consists of step 4 (stating a correct conclusion with justification). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response correctly includes the following three components:
1. States BOTH hypotheses correctly with context included in at least one of them
2. Identifies a chi-square test of independence by name or formula
3. Verifies appropriate conditions that minimally include the condition for the expected counts and do not include any incorrect conditions (such as normality)

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:
- Stating the expected count condition is not sufficient; the condition must be checked by reporting the expected counts, or minimally by showing that the smallest expected count is at least 5.
- The random sample condition was stated in the stem so need not be explicitly checked.
- If the null and alternative hypothesis are correctly stated in terms of population proportions, component 1 is satisfied. For example:
  $H_0 : p_1 = p_2 = p_3 = p_4$, where p_i is the population proportion of women at each indicated age group, 1, 2, 3, or 4, who are currently being treated for schizophrenia.
  H_a : At least one of the population proportions, p_1, p_2, p_3, p_4, differs from the other three.
  OR
  H_a : The population proportions for the four age groups are not all the same.

Section 2 is scored as follows.

Essentially correct (E) if the response correctly calculates the following two values:
1. The value of the chi-square test statistic
2. The *p*-value, critical value, or *p*-value range from chi-square table

Partially correct (P) if the response correctly calculates only one of the two values.

Question 5 (continued)

Incorrect (I) if the response does not satisfy the criteria for E or P.

Notes:

- If the response makes an error in one calculation, subsequent calculations are considered correct if they follow correctly from the initial miscalculation.
- With 3 degrees of freedom, the correct critical value is 7.81 for a significance level of 0.05 and 11.34 for a significance level of 0.01.
- Work does not have to be shown for calculations of test statistic or *p*-value. However, if incorrect work (other than minor arithmetic/transcription errors) is shown it is considered to be an incorrect calculation of the respective component, even if the correct value is given.
- If a response provides a test statistic that is not a chi-square test statistic, section 2 is scored I.

Section 3 is scored as follows.

Essentially correct (E) if the response includes the following three components:

1. A correct conclusion about the alternative hypothesis.
2. Justification of the conclusion based on linkage between the *p*-value and a reasonable alpha (or linkage between test statistic and critical value).
3. The conclusion is stated in context.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:

- If the response provides a correct decision, in context, with linkage to the *p*-value, but the decision is stated in terms of the null hypothesis with no conclusion about the alternative hypothesis, component 1 is not satisfied.
- Incorrect statistical statements are considered incorrect conclusions for the hypothesis test and do not satisfy component 1.
- If the conclusion is consistent with the *p*-value from section 2, and also in context with justification based on linkage to the *p*-value, section 3 is scored E.
- If no alpha level is given, the solution must be explicit about the linkage by giving a correct interpretation of the *p*-value or explaining how the conclusion follows from the *p*-value. For example, stating that because the *p*-value is small, we reject the null hypothesis or stating that because the *p*-value is large, we do not reject the null hypothesis.
- A decision about the null hypothesis (reject H_0 or fail to reject H_0) is not required, but if such a statement is given the scoring of the decision is considered in component 2.

Question 5 (continued)

4 Complete Response

Three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 Minimal Response

One section essentially correct

OR

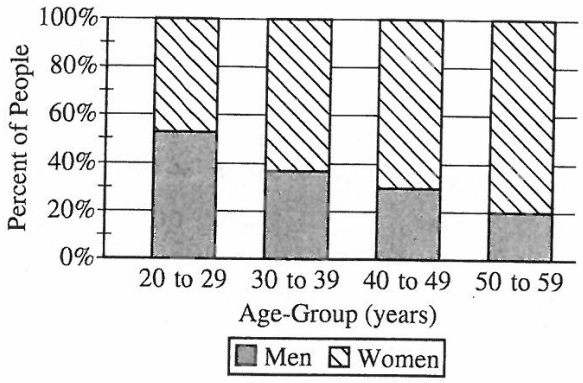
No sections essentially correct and two sections partially correct

OR

Section 1 partially correct and the other two sections incorrect

5. The table and the bar chart below summarize the age at diagnosis, in years, for a random sample of 207 men and women currently being treated for schizophrenia.

	Age-Group (years)				
	20 to 29	30 to 39	40 to 49	50 to 59	Total
Women	46	40	21	12	119
Men	53	23	9	3	88
Total	99	63	30	15	207



Do the data provide convincing statistical evidence of an association between age-group and gender in the diagnosis of schizophrenia?

State: H_0: There is not an association between age-group and gender in the diagnosis of schizophrenia

H_a: There is an association between age-group and gender in the diagnosis of schizophrenia.

plan: I will perform a chi-square test for independence if the conditions are met.

random - the sample of 207 men and women was randomly selected.

5A2

→ values in parentheses are observed; values outside of parentheses are expected

If you need more room for your work in question 5, use the space below.

	20-29	30-39	40-49	50-59	
women	56.91 (46)	36.22 (40)	17.25 (21)	8.62 (12)	
men	42.09 (53)	26.78 (23)	12.75 (9)	6.38 (3)	

LSS - as seen by the table, all expected values are at least 5.

independent - it is assumed that all observations and diagnoses are independent of each other

do: $\chi^2 = \frac{(46-56.91)^2}{56.91} + \ldots + \frac{(3-6.38)^2}{6.38} = 10.88$ $df = 3$

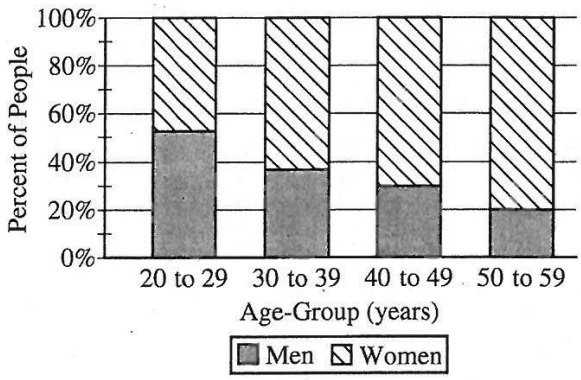
$P = 0.012$

conclude: Because the P-value is smaller than the significance level of $\alpha = 0.05$, I reject the null hypothesis. There appears to be an association between age-group and gender in the diagnosis of schizophrenia.

GO ON TO THE NEXT PAGE.

5. The table and the bar chart below summarize the age at diagnosis, in years, for a random sample of 207 men and women currently being treated for schizophrenia.

	Age-Group (years)				
	20 to 29	30 to 39	40 to 49	50 to 59	Total
Women	46	40	21	12	119
Men	53	23	9	3	88
Total	99	63	30	15	207



Do the data provide convincing statistical evidence of an association between age-group and gender in the diagnosis of schizophrenia?

State: H_0: There is no association between age-group and gender in the diagnosis of schizophrenia.

H_a: There is an association between age-group and gender in the diagnosis of schizophrenia. $\alpha = .05$

Plan: χ^2 test

Random: stated that it is a "random sample"

Independence:

Large counts: all expected counts > 5, so we can assume normality.

Expected Counts

	20-29	30-39	40-49	50-59
Women	$\frac{(119)(99)}{(207)} \approx 57$	$\frac{(119)(63)}{(207)} = 36$	$\frac{(119)(30)}{(207)} \approx 17$	$\frac{(119)(15)}{(207)} \approx 9$
Men	$\frac{(88)(99)}{(207)} \approx 42$	$\frac{(88)(63)}{(207)} \approx 27$	$\frac{(88)(30)}{(207)} \approx 13$	$\frac{(15)(88)}{(207)} \approx 6$

GO ON TO THE NEXT PAGE.

If you need more room for your work in question 5, use the space below.

Do: $df = (\#\text{ of rows} - 1)(\#\text{ of columns} - 1)$

$df = (3)(1)$ degrees of freedom $= 3$

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

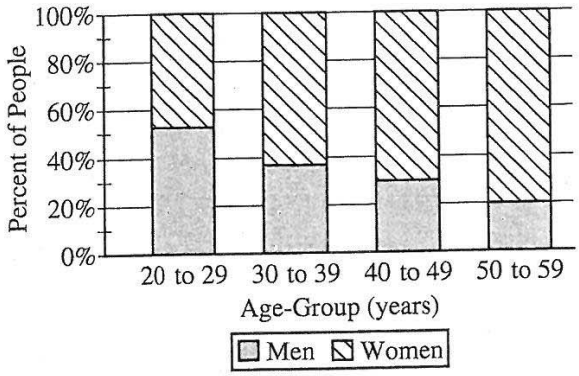
$$\chi^2 = \frac{(46-57)^2}{57} + \frac{(40-36)^2}{36} + \ldots + \frac{(9-13)^2}{13} + \frac{(3-6)^2}{6}$$

$\chi^2 \approx 10.88$ p-value $\approx .0123$

Conclude: Since the p-value $\approx .0123$ is less than $\alpha = .05$, we reject the H_0. There is sufficient evidence to support an association between age-group and gender in the diagnosis of schizophrenia.

5. The table and the bar chart below summarize the age at diagnosis, in years, for a random sample of 207 men and women currently being treated for schizophrenia.

	Age-Group (years)				
	20 to 29	30 to 39	40 to 49	50 to 59	Total
Women	46	40	21	12	119
Men	53	23	9	3	88
Total	99	63	30	15	207



Do the data provide convincing statistical evidence of an association between age-group and gender in the diagnosis of schizophrenia?

χ^2 test homogeniety

independent: one person's schizophrenia has no affect on another's schizophrenia

Random: there is a "random sample of 207 men and women"

Expected Cell Freq: the lowest expected cell frequency is 6.4

6.4 > 5

H_0: there is no association between age-group/gender in diagnosis of schizophren

H_A: there is an association between age-group/gender in diagnosis of schizophrei

GO ON TO THE NEXT PAGE.

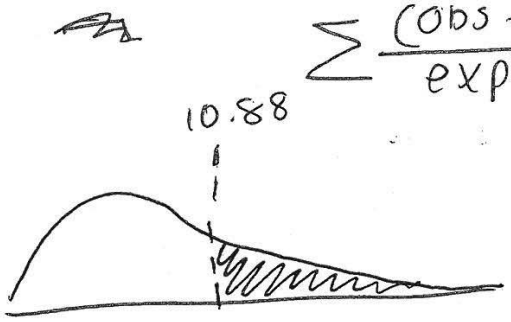
If you need more room for your work in question 5, use the space below.

$\sum \frac{(Obs - Exp)^2}{expected}$

$X^2 = 10.88$

$df = 3$

$p = .0123$



$\sum \frac{(46 - 56.913)^2}{56.913} + \ldots$

Based on this sample, with an α level of .05, and a p-value of .0123, I fail to reject the H_0, meaning I don't have evidence that there is an association between age-group/gender in the diagnosis of schizophrenia.

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2017 SCORING COMMENTARY

Question 5

Overview

The primary goal of this question was to assess a student's ability to identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question. More specific goals were to assess a student's ability to (1) state appropriate hypotheses; (2) identify the appropriate statistical test procedure and check appropriate conditions for inference; (3) calculate the appropriate test statistic and *p*-value; and (4) draw an appropriate conclusion, with justification, in the context of the study.

Sample: 5A
Score: 4

The response states the correct hypotheses in the context of the problem, which satisfies component 1 of section 1. The appropriate test procedure "chi-square test for independence" is stated; therefore, the response satisfies component 2 of section 1. The expected value condition is verified, and the expected counts are provided and clearly labeled; therefore, the response satisfies component 3 of section 1. Because the response includes the three components, section 1 was scored as essentially correct. The correct test statistic value is stated and, although work for calculating the test statistic is not required, the correct work is shown; therefore, the response satisfies component 1 of section 2. The correct *p*-value is stated in the response satisfying component 2 of section 2. Because the response includes both components, section 2 was scored as essentially correct. For a conclusion the response correctly compares the *p*-value to an appropriately chosen alpha level and, although not required, the response states the correct decision for the null hypothesis; therefore, the response satisfies component 2 of section 3. The correct conclusion about the alternative hypothesis in the context of the question is stated; therefore, components 1 and 3 of section 3 are satisfied. Because the response includes all three components, section 3 was scored as essentially correct. Because three sections were scored as essentially correct, the response earned a score of 4.

Sample: 5B
Score: 3

In section 1 the response states the appropriate hypotheses within the context of the question; therefore, the response satisfies component 1. The appropriate formula for the chi-square test of independence is stated; therefore, the response satisfies component 2 of section 1. Although the expected count condition is stated and the expected counts are listed and labeled, the response indicates the condition results in "normality," which is incorrect. The expected count condition verifies that a chi-square test statistic is appropriate. If the condition is met, it does not imply normality. The response does not satisfy component 3 of section 1. Because the response includes only two of the three components, section 1 was scored as partially correct. The response provides the correct test statistic and *p*-value. Although work was not required, the supporting work for the test statistic is correct; therefore, the response satisfies both components 1 and 2 of section 2. Because the response includes both of the components, section 2 was scored as essentially correct. The *p*-value and an appropriately chosen alpha level are correctly compared and, although it is not required, a correct decision about the null hypothesis is provided; therefore, the response satisfies component 2 of section 3. The correct conclusion about the alternative hypothesis within the context of the question is stated; therefore, the response satisfies component 1 and 3 of section 3. Because the response includes all three components, section 3 was scored as essentially correct. Because two sections were scored as essentially correct, and one section was scored as partially correct, the response earned a score of 3.

Question 5 (continued)

Sample: 5C
Score: 2

The response states the correct hypotheses within the context of the question; therefore, the response satisfies component 1 of section 1. The response incorrectly states the name of the test as a chi-square test of homogeneity. Because the question states that only one random sample is taken the appropriate hypothesis test is a chi-square test for independence. A chi-square test for homogeneity requires that within the data table either the rows or the columns are fixed, that is to say, that multiple samples are selected. The response does not satisfy component 2 of section 1. The response does verify the expected count condition by stating the lowest expected count 6.4 is greater than 5. This is enough to verify the condition; therefore, the response satisfies component 3 of section 1. Because the response includes only two of the three components, section 1 was scored as partially correct. The response states both the chi-square test statistic and the correct *p*-value. Although supporting work is not required, the supporting work provided in this response is correct. The response satisfies both components 1 and 2 of section 2. Because the response includes both components, section 2 was scored as essentially correct. Although the response indicates a comparison of the *p*-value and an appropriately chosen alpha level, the two values are never compared. The response should provide a correct comparison of those values to demonstrate a complete understanding of the justification for the conclusion to satisfy component 2. Although a decision about the null hypothesis is not required, the response provides an incorrect decision ("fail to reject the H_0"). The response does not satisfy component 2. Based on the decision stated about the null hypothesis, a correct conclusion about the alternative hypothesis is stated in the context of the question; therefore, the response satisfies component 1 and 3 of section 3. Because the response includes only two of the three components, section 3 was scored as partially correct. Because one section was scored as essentially correct, and two sections were scored as partially correct, the response earned a score of 2.