2018

AP CollegeBoard

AP Calculus AB

Sample Student Responses and Scoring Commentary

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Free Response Question 2

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AP® CALCULUS AB
2018 SCORING GUIDELINES

Question 2

(a) $v'(3) = -2.118$

The acceleration of the particle at time $t = 3$ is -2.118.

1 : answer

(b) $x(3) = x(0) + \int_0^3 v(t)\,dt = -5 + \int_0^3 v(t)\,dt = -1.760213$

The position of the particle at time $t = 3$ is -1.760.

3 : { 1 : $\int_0^3 v(t)\,dt$; 1 : uses initial condition; 1 : answer }

(c) $\int_0^{3.5} v(t)\,dt = 2.844$ (or 2.843)

$\int_0^{3.5} |v(t)|\,dt = 3.737$

The integral $\int_0^{3.5} v(t)\,dt$ is the displacement of the particle over the time interval $0 \le t \le 3.5$.

The integral $\int_0^{3.5} |v(t)|\,dt$ is the total distance traveled by the particle over the time interval $0 \le t \le 3.5$.

3 : { 1 : answers; 2 : interpretations of $\int_0^{3.5} v(t)\,dt$ and $\int_0^{3.5} |v(t)|\,dt$ }

(d) $v(t) = x_2'(t)$

$v(t) = 2t - 1 \Rightarrow t = 1.57054$

The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).

2 : { 1 : sets $v(t) = x_2'(t)$; 1 : answer }

2A
1 of 2

2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

$v'(3) = -2.1182$

(b) Find the position of the particle at time $t = 3$.

$-5 + \int_0^3 v(t)\,dt = -1.7602$

Continue question 2 on page 7.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

$\int_0^{3.5} v(t)\,dt = 2.8439$, which is the displacement of the particle from $t=0$ to $t=3.5$

$\int_0^{3.5} |v(t)|\,dt = 3.7371$, which is the total distance the particle traveled from $t=0$ to $t=3.5$

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$v(t) = 2t - 1$

$t = 1.5705$

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2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

$$v(t) = \frac{10\sin(0.4t^2)}{t^2 - t + 3}$$

$$v'(3) = -2.11819$$

(b) Find the position of the particle at time $t = 3$.

$$-5 + \int_0^3 \frac{10\sin(0.4t^2)}{t^2 - t + 3}\,dt = -1.760$$

Continue question 2 on page 7.

2 2 2 2 2 2 2 2 2 2

2B, 2.62

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

$\int_0^{3.5} v(t)\,dt = 2.843$

$\int_0^{3.5} v(t)$ means the position of the particle at time $t = 3.5$ which is 2.843 units.

$\int_0^{3.5} |v(t)| = 3.737$

$\int_0^{3.5} |v(t)|$ means the total distance that the particle has traveled from $t = 0$ to $t = 3.5$

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$x_2(t) = t^2 - t$

$v_2(t) = 2t\ dt$

There is not a time where the two particles are moving with the same velocity because their velocity functions never intersect with each other.

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2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

$$a(t) = v'(t) = \frac{(2t-1)(10\sin(0.4t^2)) - 8t(t^2-t+3)(10\cos(0.4t^2))}{(t^2-t+3)^2}$$

$$a(3) = \frac{(6-1)(10\sin(3.6)) - 7.4(9-3+3)(10\cos(3.6))}{(9-3+3)^2}$$

$$= \frac{50\sin(3.6) - 216.\cos(3.6)}{81}$$

$$= 2.118$$

(b) Find the position of the particle at time $t = 3$.

$$P = \int_0^3 v(t)\,dt = \int_0^3 \frac{10\sin(0.4t^2)}{t^2-t+3}\,dt = 3.240 \text{ m}$$

Continue question 2 on page 7.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

$\int_0^{3.5} v(t)dt = \int_0^{3.5} \frac{10\sin(0.4t^2)}{t^2-t+3}dt = 2.844$ m → the displacement of the particle

$\int_0^{3.5} |v(t)|dt = \int_0^{3.5} \left|\frac{10\sin(0.4t^2)}{t^2-t+3}\right|dt = 3.737$

→ the distance traveled by the particle

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$t^2 - t = \frac{10\sin(0.4t^2)}{t^2-t+3}$

$t = 0,\ 2.000$

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AP® CALCULUS AB
2018 SCORING COMMENTARY

Question 2

Overview

In this problem a particle moves along the x-axis. For $0 \le t \le 3.5$, the velocity of the particle is given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$, and the particle's position is $x = -5$ at time $t = 0$. In part (a) students were asked for the acceleration of the particle at time $t = 3$. A correct response should demonstrate that acceleration is the derivative of velocity and show the evaluation of $v'(3)$ from a graphing calculator. In part (b) students were asked for the position of the particle at time $t = 3$. A correct response should find the net change in the particle's position as the integral of $v(t)$ across the interval $[0, 3]$ and add this change in position to the particle's position at time $t = 0$. In part (c) students were asked to evaluate the integrals $\int_0^{3.5} v(t)\,dt$ and $\int_0^{3.5} |v(t)|\,dt$ and to interpret the meaning of each integral in the context of the problem. A correct response should show the values of the two integrals obtained from a graphing calculator and convey that a definite integral of velocity gives the particle's displacement, while a definite integral of speed (i.e., $|v(t)|$) gives the particle's total distance traveled, across the time interval of integration. In part (d) students were given that a second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$ and are asked for the time t when the two particles are moving with the same velocity. A correct response should demonstrate that the second particle's velocity is obtained by differentiating its position function and proceed by solving for when the first particle's velocity, the given $v(t)$, matches $x_2'(t)$ within the interval $0 \le t \le 3.5$.

For part (a) see LO 2.3C/EK 2.3C1. For parts (b) and (c), see LO 3.3B(b)/EK 3.3B2, LO 3.4C/EK 3.4C1. For part (d) see LO 2.3C/EK 2.3C1. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, building notational fluency, and communicating.

Sample: 2A
Score: 9

The response earned all 9 points: 1 point in part (a), 3 points in part (b), 3 points in part (c), and 2 points in part (d). In part (a) the response earned the point with the equation because it gives the correct answer along with the identification of this value as $v'(3)$. In part (b) the response earned the first point with the definite integral on the left side of the equation. The response earned the second point with the addition of -5 to that integral. The response earned the third point with the answer on the right side of the equation. In part (c) the response earned the first point with the values of the two definite integrals. The response earned both of the interpretation points as the two integrals are interpreted as "displacement" and "total distance," and both reference the time interval. In part (d) the response earned the first point in line 1 because the right side of the equation is an expression equivalent to $x_2'(t)$. The answer $t = 1.5705$ earned the second point.

Sample: 2B
Score: 6

The response earned 6 points: 1 point in part (a), 3 points in part (b), 2 points in part (c), and no points in part (d). In part (a) the response earned the point with the equation in line 2 because it identifies the correct answer as $v'(3)$. In part (b) the response earned the first point with the definite integral on the left side of the equation. The response earned the second point with the addition of -5 to that integral. The response earned the third point with

Question 2 (continued)

the answer on the right side of the equation. In part (c) the response earned the first point with the values of the two definite integrals. The interpretation of the first integral as "the position of the particle at time $t = 3.5$" is incorrect. The interpretation of the second integral as "the total distance that the particle has traveled from $t = 0$ to $t = 3.5$" is correct. Thus, the response earned 1 of the 2 interpretation points in part (c). In part (d) the response did not earn the first point because $x_2'(t)$ is not set equal to $v(t)$. The response also incorrectly indicates "there is not a time where the two particles are moving with the same velocity," so the answer point was not earned.

Sample: 2C
Score: 3

The response earned 3 points: no point in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d). In part (a) the response has an attempt to use the quotient rule to find $v'(3)$. However, the two quantities in the numerator of the stated derivative are reversed and result in the negative of $v'(t)$. This error produces an incorrect value of 2.118, so the point was not earned. In part (b) the response earned the first point with the integral $\int_0^3 v(t)\,dt$. Because there is no use of the initial condition, the answer is incorrect, and no other points in part (b) were earned. In part (c) the response earned the first point with the values of the two definite integrals. The identifications of the first integral as displacement and the second integral as distance traveled are correct, but the response does not reference the time interval in either case. The response earned 1 of the 2 interpretation points in part (c). In part (d) the response has an incorrect equation that leads to an incorrect answer, so no points were earned.