Chief Reader Report on Student Responses:

2018 AP® Calculus AB and Calculus BC Free-Response Questions

Number of Readers (Calculus AB/Calculus BC):	1,051		
Calculus AB			
• Number of Students Scored	308,538		
• Score Distribution	Exam Score	N	%At
	5	59,733	19.4
	4	53,255	17.3
	3	64,768	21.0
	2	68,980	22.4
	1	61,802	20.0
• Global Mean	2.94		
Calculus BC			
• Number of Students Scored	139,376		
• Score Distribution	Exam Score	N	%At
	5	56,324	40.4
	4	25,982	18.6
	3	28,891	20.7
	2	20,349	14.6
	1	7,830	5.6
• Global Mean	3.74		
Calculus BC Calculus AB Subscore			
• Number of Students Scored	139,376		
• Score Distribution	Exam Score	N	%At
	5	67,859	48.7
	4	28,129	20.2
	3	22,184	15.9
	2	13,757	9.9
	1	7,447	5.3
• Global Mean	3.97		

The following comments on the 2018 free-response questions for AP® Calculus AB and Calculus BC were written by the Chief Reader, Stephen Davis of Davidson College. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student preparation in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question AB1/BC1 **Topic:** Modeling Rate

Max. Points: 9 **Mean Score: AB1:** 2.82; **BC1:** 4.13

What were the responses to this question expected to demonstrate?

The context of this problem is a line of people waiting to get on an escalator. The function r models the rate at which people enter the line, where $r(t) = 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7$ for $0 \le t \le 300,$ and $r(t) = 0$ for $t > 300;$ $r(t)$ is measured in people per second, and t is measured in seconds. Further, it is given that people exit the line to get on the escalator at a constant rate of 0.7 person per second and that there are 20 people in the line at time $t = 0.$ In part (a) students were asked how many people enter the line for the escalator during the time interval $0 \le t \le 300.$ A correct response demonstrates the understanding that the number of people entering the line during this time interval is obtained by integrating the rate at which people enter the line across the time interval. Thus, this number is the value of the definite integral $\int_0^{300} r(t)\, dt.$ A numerical value for this integral should be obtained using a graphing calculator. In part (b) students were given that there are always people in line during the time interval $0 \le t \le 300$ and were asked to determine the number of people in line at time $t = 300.$ A correct response should take into account the 20 people in line initially, the number that entered the line as determined in part (a), and the number of people that exit the line to get on the escalator. It was given in the problem statement that people exit the line at a constant rate of 0.7 person per second, so the number of people that exit the line to get on the escalator can be found by multiplying this constant rate times the duration of the interval, namely 300 seconds. In part (c) students were asked for the first time t beyond $t = 300$ when there are no people in line for the escalator. Because no more people join the line after $t = 300$ seconds, and people exit the line at the constant rate of 0.7 person per second, dividing the answer to part (b) by 0.7 gives the number of seconds beyond $t = 300$ before the line empties for the first time. Adding this quotient to 300 produces the answer. In part (d) students were asked when, during the time interval $0 \le t \le 300,$ is the number of people in line a minimum, and to determine the number of people in line (to the nearest whole number) at that time, with the added admonition to justify their answer. The Extreme Value Theorem guarantees that the number of people in line at time $t,$ given by the expression $20 + \int_0^t r(x)\, dx - 0.7t,$ attains a minimum on the interval $0 \le t \le 300.$ Correct responses should demonstrate that the rate of change of the number of people in line is given by $r(t) - 0.7.$ Solving for $r(t) - 0.7 = 0$ within the interval $0 < t < 300$ yields two critical points, t_1 and t_2, so candidates for the time when the line is a minimum are $t = 0,$ $t_1,$ $t_2,$ and $t = 300.$ The number of people in line at times t_1 and t_2 is computed from $20 + \int_0^{t_1} r(x)\, dx - 0.7t_1$ and $20 + \int_0^{t_2} r(x)\, dx - 0.7t_2.$ The answer is the least of 20, these two computed values (to the nearest whole number), and the answer to part (b), together with the corresponding time t for this minimum value.

For part (a) see LO 3.3B(b)/EK 3.3B2, LO 3.4A/EK 3.4A2, LO 3.4E/EK 3.4E1. For parts (b) and (c), see LO 3.4A/EK 3.4A2, LO 3.4E/EK 3.4E1. For part (d) see LO 1.2B/EK 1.2B1, LO 2.3C/EK 2.3C3, LO 3.3B(b)/EK 3.3B2, LO 3.4A/EK 3.4A2, LO 3.4E/EK 3.4E1. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, building notational fluency, and communicating.

How well did the response address the course content related to this question? How well did the responses integrate the skills required on this question?

In part (a), many responses showed understanding of the need to integrate a rate of change to find the net change in the number of people entering the line for the escalator. Some included the rate at which people left the line as part of the integrand, and some revealed a misunderstanding of the question by including the number of people in line initially in the answer. In part (b), responses generally accounted in some way for the rate that people left the line, although some failed to account for the 20 people in the line initially. In part (d), responses implied challenges in recognizing an objective function to minimize, evidenced by the use of $r(t)$ or $r'(t)$ for the rate of change (derivative) of the number of people in line instead of the use of $r(t) - 0.7.$ Some responses identified the correct time for a minimum value, but fell short in justification of the minimum value. This included justifications that were incomplete by not considering the second critical point and/or not identifying the specific minimum value.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• In part (b), omitting a differential could render an integral incorrect, as in $\int_0^{300} r(t) - 0.7 + 20,$ which could be interpreted as $\int_0^{300} r(t)\, dt - 0.7 + 20$ OR $\int_0^{300} (r(t) - 0.7 + 20)\, dt.$	• The number of people in line at time $t = 300$ is given by $20 + \int_0^{300} (r(t) - 0.7)\, dt.$
• In part (c), responses indicating the line has no people at $t = \frac{80}{0.7} = 114.286$ seconds suggest a misunderstanding of how to handle the two time frames in the piecewise defined function r.	• The first time that there are no people in line is at time $t = 300 + \frac{80}{0.7} = 414.286$ seconds.
• In part (d), responses identifying the critical points as solutions to $r(t) = 0$ suggest a misinterpretation of the appropriate function to be minimized.	• The number of people in line at time t is given by $p(t) = 20 + \int_0^{t} (r(x) - 0.7)\, dx.$ Critical points for the function are solutions to $p'(t) = r(t) - 0.7 = 0.$
• In part (d), basing a conclusion on only one zero of $r(t) - 0.7$ may represent a misconception of how to justify an absolute minimum using the candidates test (e.g., $r(t) - 0.7 = 0 \Rightarrow t = 33.0133 \Rightarrow$ The minimum is 3.803 at $t = 33.013.$)	• $r(t) - 0.7 = 0 \Rightarrow t_1 = 33.0133,\ t_2 = 166.5747$ $r(0) = 20,\ r(t_1) = 3.803,\ r(t_2) = 158.070,$ $r(300) = 80.$ Therefore, the minimum is 4 people at time $t = 33.013$ seconds.

Based on your experience at the AP® Reading with student responses, what advice would you offer to teachers to help them improve the student performance on the exam?

Teachers can emphasize careful reading to distinguish, for example, the number of people that enter a line during a specified period from the number of people in line at the end of that period. One of these needs to consider the number in line at the start of the time period, while the other does not. Careful reading can also help students to make sure that they answer the specific question asked as opposed to a answering a generic problem. Does the question ask for the time the line is shortest, the number in line when it is shortest, or both?

Teachers should continue to emphasize the appropriate use of a differential to close an integral expression. With respect to graphing calculator use, teachers can coach students to store intermediate results in their calculator. This retains greater accuracy, whereas rounding intermediate results to 3 or fewer decimal places may produce a final answer that is not accurate to the required three decimal places.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- The *AP Calculus AB and Calculus BC Course and Exam Description* (*CED*) includes instructional resources for AP Calculus teachers to develop students' broader skills. Please see page 28 of the *CED* for examples of MPACs, questioning, and instructional strategies designed to develop the broader skill of "Justification," which was important in this question. A table of representative instructional strategies, including definitions and explanations of each, is included on pages 33–37 of the *CED*. The strategy "Critique reasoning," for example, may be helpful in developing students' reasoning and communication skills at multiple points across the curriculum, including when learning to use a candidates test to justify a conclusion about an absolute minimum value.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include a "Classroom Resources" tab, where you will find a variety of teaching modules developed by the College Board over the years. For this question, you might refer to the module *Extrema*, which includes commentary, related previous AP Exam questions, student worksheets, teacher notes, and teaching examples.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include "The Exam" tab, where you (and your students) will find questions from previous AP Exams and reflections of chief readers. In addition to end-of-course review, these resources are extremely useful for low-stakes, formative assessment, from which you may base high-quality feedback and responsive instruction. Careful study of 2017 question AB2 part (a), for example, would have been excellent preparation for 2018 question AB1/BC1 part (a). Consistently holding students accountable for clear mathematical communication, including attention to parentheses and placement of differentials, is the best way to develop such skills.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include a "Professional Development" tab, where you will find:
    - Links to Online Modules for Teaching and Assessing AP Calculus. The module *Justifying Properties and Behaviors of Functions Using Derivatives* would be relevant to this question.
    - A link to Davidson Next, an initiative that, "aims to supplement Advanced Placement (AP) instruction with online modules designed for in-class, blended instruction."
- Finally, the Online Teacher Community is a great place to ask a question, share a strategy, or hear from other AP teachers.

Question AB2 **Topic:** Particle Motion

Max. Points: 9 **Mean Score:** 4.21

What were the responses to this question expected to demonstrate?

In this problem a particle moves along the x-axis. For $0 \le t \le 3.5,$ the velocity of the particle is given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3},$ and the particle's position is $x = -5$ at time $t = 0.$ In part (a) students were asked for the acceleration of the particle at time $t = 3.$ A correct response should demonstrate that acceleration is the derivative of velocity and show the evaluation of $v'(3)$ from a graphing calculator. In part (b) students were asked for the position of the particle at time $t = 3.$ A correct response should find the net change in the particle's position as the integral of $v(t)$ across the interval $[0, 3]$ and add this change in position to the particle's position at time $t = 0.$ In part (c) students were asked to evaluate the integrals $\int_0^{3.5} v(t)\,dt$ and $\int_0^{3.5} |v(t)|\,dt$ and to interpret the meaning of each integral in the context of the problem. A correct response should show the values of the two integrals obtained from a graphing calculator and convey that a definite integral of velocity gives the particle's displacement, while a definite integral of speed (i.e., $|v(t)|$) gives the particle's total distance traveled, across the time interval of integration. In part (d) students were given that a second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$ and are asked for the time t when the two particles are moving with the same velocity. A correct response should demonstrate that the second particle's velocity is obtained by differentiating its position function and proceed by solving for when the first particle's velocity, the given $v(t),$ matches $x_2{}'(t)$ within the interval $0 \le t \le 3.5.$

For part (a) see LO 2.3C/EK 2.3C1. For parts (b) and (c), see LO 3.3B(b)/EK 3.3B2, LO 3.4C/EK 3.4C1. For part (d) see LO 2.3C/EK 2.3C1. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, building notational fluency, and communicating.

How well did the response address the course content related to this question? How well did the responses integrate the skills required on this question?

In part (a), most responses gave evidence of an intent to differentiate velocity to obtain acceleration. Failure to earn this point was due to errors in entering the correct velocity expression in the graphing calculator, calculations in degree mode, not communicating a derivative setup in the presence of a correct numerical answer, or errors in symbolic attempts at differentiating the velocity function. Similarly, in part (b), many responses showed computation of the required definite integral, and many of those also accounted for the particle's initial position in the answer. Some responses had decimal presentation errors (answers not accurate to the required three decimal places). Again, errors in entry of the velocity expression in the graphing calculator could impact this part. In part (c), responses generally gave correct numerical values for the requested integrals, and could identify the first integral as giving displacement and the second integral as giving total distance traveled. Some omitted reference to the required time interval in the interpretations or erroneously used and referenced 3 instead of 3.5 as the upper limit of the definite integrals. In part (d), responses revealed efforts at equating $v(t)$ with the derivative of $x_2(t),$ although some responses omitted showing the required equation setup, and other responses used $2t$ instead of $2t - 1$ for $x_2{}'(t).$

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• In part (b), omitting the initial condition to represent the position of the particle at time $t = 3$ as $\int_0^3 v(t)\,dt$	• The position of the particle at time $t = 3$ is $-5 + \int_0^3 v(t)\,dt.$
• In part (c), omitting a time reference in the interpretations of $\int_0^{3.5} v(t)\,dt$ and $\int_0^{3.5} \lvert v(t)\rvert\,dt$ or interpreting the integrals as behavior at time $t = 3.5$	• $\int_0^{3.5} v(t)\,dt$ and $\int_0^{3.5} \lvert v(t)\rvert\,dt$ are displacement and total distance traveled, respectively, for the particle from time $t = 0$ to time $t = 3.5.$

Based on your experience at the AP® Reading with student responses, what advice would you offer to teachers to help them improve the student performance on the exam?

Some responses showed the need for more practice with graphing calculator skills. For example, the lack of parentheses when entering the denominator for $v(t)$ could result in entering the expression $\frac{10\sin\left(0.4t^2\right)}{t^2} - t + 3,$ resulting in incorrect numerical answers throughout. Also, students should be reminded to show the mathematical setup (the function, integral, or equation) for graphing calculator evaluation of numerical derivatives, definite integrals, and solutions to an equation. The distinction between complete and incomplete mathematical communication can be subtle. For example, labeling the answer in part (a) as $a(3)$ does not fully communicate that acceleration is the derivative of velocity, while labeling the answer as $v'(3)$ does.

Teachers can continue to work on student communication, both through appropriate presentation of symbolic mathematics and in interpretations. Poor presentations (e.g., misuse of the equals sign, a missing differential, or a run-on computation string) gives incorrect mathematics and can lead to confusion for the student. A complete interpretation of an expression such as $\int_0^{3.5} v(t)\,dt$ should involve all aspects of the expression, not just an interpretation of the antiderivative of a velocity function but also the role of the limits of integration.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- The *AP Calculus AB and Calculus BC Course and Exam Description* (*CED*) includes instructional resources for AP Calculus teachers to develop students' broader skills. Please see page 32 of the *CED* for examples of MPACs, questioning, and instructional strategies designed to develop the broader skill of "Application," which was important in this question. A table of representative instructional strategies, including definitions and explanations of each, is included on pages 33–37 of the *CED*. The strategies "Notation read aloud" and/or "Error analysis," for example, may be helpful in developing students' reasoning and communication skills at multiple points across the curriculum, including when learning to fully and clearly communicate setups for computations performed by the graphing calculator and to accurately enter expressions into the graphing calculator.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include a "Classroom Resources" tab, where you will find a variety of teaching modules developed by the College Board over the years. For this question, you might refer to the module *Motion*, which includes commentary, related previous AP Exam questions, student worksheets, teacher notes, and teaching examples.

- AP Central's course pages for AP Calculus AB and AP Calculus BC include "The Exam" tab, where you (and your students) will find questions from previous AP Exams and reflections of chief readers. In addition to end-of-course review, these resources are extremely useful for low-stakes, formative assessment, from which you may base high quality feedback and responsive instruction. Multiple-choice questions 6, 16, 28, 79, 83, and 89 from the released 2012 Calculus AB exam offer opportunities for formative assessment and feedback on particle motion. Although 2017 question AB5 asks different particle motion questions from 2018 question AB2, the 2017 question is an excellent resource for fully developing concepts related to particle motion.
- As you practice to develop skillful and appropriate use of graphing calculators, you may wish to refer students to the AP Calculus AB "Essential Exam Tips" page, where they will find the following advice:

> Show your work, even when you're using a calculator. Show **all** the steps you took to reach your solution on questions involving calculations. The exam reader wants to see if you know how to solve the problem. If you use your calculator to solve an equation, compute a numerical derivative, or find a definite integral, then be sure to write the equation, derivative, or integral first: an answer without this information might not get full credit, even if the answer is correct. Remember to write your work in standard notation (e.g., $\int_1^5 x^2\, dx$) rather than calculator syntax (e.g., fnInt(X^2,X,1,5)), as calculator syntax is not acceptable.

- Finally, the Online Teacher Community is a great place to ask a question, share a strategy, or hear from other AP teachers.

Question AB3/BC3 **Topic:** Graphical Analysis of *f* prime/FTC

Max. Points: 9 **Mean Score: AB3:** 3.36; **BC3:** 5.18

What were the responses to this question expected to demonstrate?

In this problem the graph of the continuous function g is provided; g is piecewise linear for $-5 \le x < 3,$ and $g(x) = 2(x-4)^2$ for $3 \le x \le 6.$ It is also given that g is the derivative of the function *f*. In part (a) students were given that $f(1) = 3$ and asked for the value of $f(-5).$ A correct response should demonstrate knowledge that *f* is an antiderivative of *g*, so that $f(-5) = f(1) + \int_1^{-5} g(x)\,dx.$ The integral $\int_1^{-5} g(x)\,dx$ should then be evaluated using properties of definite integrals and computation of areas of the regions between the graph of *g* and the *x*-axis using geometry. In part (b) students were asked to evaluate $\int_1^6 g(x)\,dx.$ A correct response should use the property of integrals to split the interval of integration into the sum of integrals across adjacent intervals $[1, 3]$ and $[3, 6].$ One of the resulting integrals can be computed using geometry and the other using an antiderivative of $g(x) = 2(x-4)^2$ on the interval $3 \le x \le 6.$ In part (c) students were asked for the open intervals on $-5 < x < 6$ where the graph of *f* is both increasing and concave up and to give a reason for their answer. A correct response should demonstrate the connection between properties of the derivative of *f* and the properties of monotonicity and concavity for the graph of *f*. The graph of *f* is strictly increasing where $g = f'$ is positive, and the graph of g is concave up where the graph of $g = f'$ is increasing. In part (d) students were asked for the *x*-coordinate of each point of inflection of the graph of *f* and to give a reason for their answer. A correct response should convey that a point of inflection of the graph of *f* occurs at a point where the derivative of *f* changes from increasing to decreasing, or from decreasing to increasing. This can be obtained from the supplied graph of $g = f',$ which changes from decreasing to increasing at $x = 4.$

For part (a) see LO 3.2C/EK 3.2C1, LO 3.2C/EK 3.2C2. For part (b) see LO 3.2C/EK 3.2C1, LO 3.2C/EK 3.2C2, LO 3.3B(b)/EK 3.3B2, LO 3.3B(b)/EK 3.3B5. For parts (c) and (d), see LO 2.2A/EK 2.2A1. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating.

How well did the response address the course content related to this question? How well did the responses integrate the skills required on this question?

In part (a), many responses attempted to deal with $\int_1^{-5} g(x)\,dx$ or $\int_{-5}^{1} g(x)\,dx.$ Students' responses were often challenged with trying to evaluate the integral using geometry, however, or could not complete an analytic attempt. Some responses treated $\int_1^{-5} g(x)\,dx$ as the value of $f(-5)$ as opposed to the net change in *f* from $f(1) = 3$ to $f(-5).$ In part (b), some responses dealt with splitting the integral at $x = 3,$ but mistakenly computed $\int_1^{3} g(x)\,dx$ as 5, perhaps by integrating from $x = 0$ instead of from $x = 1.$ Some responses entered with $5 + \int_3^6 g(x)\,dx,$ casting doubt that an appropriate split at $x = 3$ had been attempted. In part (c), instead of reasoning from the behavior of the graph of $g = f',$ some responses attempted to reason from the sign of f' without explicitly connecting f' to the graph of *g*. Some responses had only the interval $(4, 6),$ perhaps answering instead for where the graph of g is both increasing and concave up. In part (d), many responses used incomplete reasoning that a zero for f' guarantees a point of inflection at the corresponding location on the graph of *f*. In both parts (c) and (d), some responses indicated uncertainty on how to deal with intervals on which $g = f'$ is constant.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• In part (a), omitting the initial condition: $f(-5) = \int_1^{-5} g(x)\,dx = -1 + \frac{3}{2} + 9$	• $f(-5) = f(1) + \int_1^{-5} g(x)\,dx = 3 - 1 + \frac{3}{2} + 9$
• In part (b), extending the expression $2(x-4)^2$ to describe $g(x)$ for values of x less than 3: $\int_1^6 g(x)\,dx = \int_1^6 2(x-4)^2\,dx = \left[\frac{2}{3}(x-4)^3\right]_{x=1}^{x=6} = \frac{16}{3} - (-18)$	• $\int_1^6 g(x)dx = \int_1^3 g(x)\,dx + \int_3^6 2(x-4)^2\,dx = 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right)$
• In part (c), confusing the formula for a piece of the graph of g as applying to *f*: the graph of *f* is increasing and concave up on the interval $(4, 6)$ because $y = 2(x-4)^2$ is increasing and concave up there.	• The graph of *f* is increasing and concave up on the intervals $(0, 1)$ and $(4, 6)$ because (from the graph of *g*) $f'(x) = g(x)$ is positive and the graph of $f' = g$ is increasing on these intervals.
• In part (d), making an incomplete argument for a point of inflection that fails to consider the sign change in $f''(x)$ at $x = 4$: the graph of *f* has a point of inflection at $x = 4$ because $f''(4) = g'(4) = 4(x-4)\Big\vert_{x=4} = 0$.	• The graph of *f* has a point of inflection at $x = 4$ because the graph of $f' = g$ changes from decreasing to increasing at $x = 4$. (OR because $f''(x) = g'(x) = 4(x-4)$ changes from negative to positive at $x = 4$.)

Based on your experience at the AP® Reading with student responses, what advice would you offer to teachers to help them improve the student performance on the exam?

Students can use more practice working from functions for which the presentation is mixed; a portion of the graph consists of linear segments, and another portion is given analytically. If the presented function (here, *g*) is the derivative of another function *f* (so $g = f'$), then determining values of *f* requires computing definite integrals with a mixture of geometric and analytic methods. Further, teachers can emphasize that a definite integral of f' gives a net change in values of *f* from the lower limit to the upper limit of the integral. That is, as we know from the Fundamental Theorem of Calculus, computation of $\int_a^b f'(x)\,dx$ gives $f(b) - f(a)$, not $f(b)$. To determine $f(b)$ from this requires adding $f(a)$, thus $f(b) = f(a) + \int_a^b f'(x)\,dx$. Students can benefit from practice in this area.

Also, when asked about concavity of the graph of a function *f*, many students default to looking only at the second derivative, f''. Teachers can encourage students to use information about f' to resolve issues of concavity for the graph of *f* when such information is readily available.

Finally, teachers can work on refining students' communication skills to provide justifications where the connection to given information is clearly communicated. In this case, we were given a graph of g and asked questions about a function *f*. Justifications of properties of *f* (e.g., where the graph of *f* is increasing and concave up) need to reference g to go beyond the appearance of being formulaic.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- The *AP Calculus AB and Calculus BC Course and Exam Description* (*CED*) includes instructional resources for AP Calculus teachers to develop students' broader skills. Please see page 29 of the *CED* for examples of MPACs, questioning, and instructional strategies designed to develop the broader skill of "Modeling," which was evident in the area of multiple representations in this question. A table of representative instructional strategies, including definitions and explanations of each, is included on pages 33–37 of the *CED*. The strategies "Create representations" and/or "Marking the text," for example, may be helpful in developing students' skills at extracting key and relevant information at multiple points across the curriculum, including when learning to draw conclusions about the behavior of a function based on a graph of its derivative, as in this question.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include a "Classroom Resources" tab, where you will find an excellent resource *The Fundamental Theorem of Calculus* under the heading "Special Focus Materials." You will also find a variety of teaching modules developed by the College Board over the years. For this question, you might refer to the module *Fundamental Theorem of Calculus*, which includes commentary, related previous AP Exam questions, student worksheets, teacher notes, and teaching examples.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include "The Exam" tab, where you (and your students) will find questions from previous AP Exams and reflections of chief readers. In addition to end-of-course review, these resources are extremely useful for low-stakes, formative assessment, from which you may base high-quality feedback and responsive instruction. Question AB3/BC3 from the 2017 exams and the corresponding chief reader notes would have been very helpful in developing the concepts and skills necessary to succeed on question AB3/BC3 on the 2018 exams.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include the "Professional Development" tab, where you will find:
    - Links to Online Modules for Teaching and Assessing AP Calculus. The module *Justifying Properties and Behaviors of Functions Using Derivatives* would be relevant to this question.
    - A link to Davidson Next, an initiative that, "aims to supplement Advanced Placement (AP) instruction with online modules designed for in-class, blended instruction."
- Finally, the Online Teacher Community is a great place to ask a question, share a strategy, or hear from other AP teachers.

Question AB4/BC4 **Topic:** Modeling/Related Rates-Tabular/Analytic

Max. Points: 9 **Mean Score: AB4:** 2.77; **BC4:** 4.43

What were the responses to this question expected to demonstrate?

The context of this problem is a tree, the height of which at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are provided in a table. In part (a) students were asked to use the tabular data to estimate $H'(6)$ and then to interpret the meaning of $H'(6)$, using correct units, in the context of the problem. The correct response should estimate the derivative value using a difference quotient, drawing from data in the table that most tightly bounds $t = 6$. In part (b) students were asked to explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$. A correct response should demonstrate that the Mean Value Theorem applies to H on the interval $[3, 5]$, over which the average rate of change of H (using data from the table) is $\frac{6-2}{2} = 2$. In part (c) students were asked to use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$. A correct response should demonstrate that the average height of the tree for $2 \le t \le 10$ is given by dividing the definite integral of H across the interval by the width of the interval. The value of the integral $\int_2^{10} H(t)\,dt$ is to be approximated using a trapezoidal sum and data in the table. In part (d) students were given another model for the tree's height, in meters, $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. It is further given that when the tree is 50 meters tall, it is growing so that the diameter at the base of the tree is increasing at the rate of 0.03 meter per year. Using this model, students were asked to find the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall. A correct response should apply the chain rule to obtain that $\frac{dG}{dt} = \frac{dG}{dx} \cdot \frac{dx}{dt}$. The derivative expression $\frac{dG}{dx}$ can be obtained from the given expression for $G(x)$ using derivative rules (e.g., the quotient rule) and the value of $\frac{dx}{dt}$ at the instant in question provided in the problem statement.

For part (a) see LO 2.1B/EK 2.1B1, LO 2.3A/EK 2.3A1, LO 2.3A/EK 2.3A2. For part (b) see LO 2.4A/EK 2.4A1. For part (c) see LO 3.2B/EK 3.2B2, LO 3.4B/EK 3.4B1. For part (d) see LO 2.1C/EK 2.1C3, LO 2.1C/EK 2.1C4, LO 2.3C/EK 2.3C2. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating.

How well did the response address the course content related to this question? How well did the responses integrate the skills required on this question?

In part (a), responses generally used a difference quotient to approximate $H'(6)$, although some difference quotients were over the interval $[3, 7]$ instead of $[5, 7]$. Although correct units were often reported, many responses gave either no or an incomplete interpretation of $H'(6)$. The interpretation required three necessary elements; an interpretation of H' as a rate in the context of the tree, correct units of meters per year, and an interpretation of the input "6" as the moment in time of year 6. An incorrect description for the rate or failing to reference the "6" were common. In part (b), many responses investigated the Mean Value Theorem as possibly relevant, but often did not identify an appropriate interval on which to obtain a difference quotient equal to 2. Explanations often omitted or gave incomplete conditions to apply the Mean Value Theorem. In particular, it was very common to omit mention of the continuity hypothesis. Because H was given to be twice differentiable, it was possible to given an alternative argument: use the Mean Value Theorem on two subintervals to obtain existence of values $t = t_1$ and $t = t_2$ for which $H'(t_1)$ and $H'(t_2)$ bracket 2, and then apply the Intermediate

Value Theorem to the interval between t_1 and t_2. A few responses attempted this difficult and complicated argument but rarely with success. In part (c), many responses successfully employed a trapezoidal sum to approximate $\int_2^{10} H(t)\,dt$, although some gave evidence of unsuccessful attempts to use an algorithm that assumed a regular partition of $[2, 10]$. Many responses failed to take the final step of dividing the integral approximation by 8 to approximate the average height. In part (d), responses generally showed facility with the quotient rule (or the product rule in an alternate approach), and many responses also correctly applied the chain rule. However, responses were often defeated by notational errors or confusion between variables.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• In part (b), after noting that $\frac{H(5) - H(3)}{5 - 3} = 2$, jumping straight to conclude that $H'(t) = 2$ has a solution in the interval $(3, 5)$ by the Mean Value Theorem, without affirming the hypotheses of the theorem	• $\frac{H(5) - H(3)}{5 - 3} = 2$ Because H is differentiable on $[3, 5]$, H is continuous on that interval. Thus, the Mean Value Theorem guarantees a value c with $3 < c < 5$ and $H'(c) = 2$.
• In part (c), using a formula for trapezoidal approximation of an integral that assumes a regular partition: $\int_2^{10} H(x)\,dx$ $\approx \frac{10 - 2}{4} \cdot \frac{1}{2} \cdot (1.5 + 2 \cdot 2 + 2 \cdot 6 + 2 \cdot 11 + 15)$ $= 54.5$	• The correct trapezoidal approximation is $\int_2^{10} H(x)\,dx$ $\approx \frac{1.5 + 2}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot 3$ $= 65.75.$
• In part (d), after deriving that $\frac{dG}{dx} = \frac{100}{(1 + x)^2}$, using 50 as a value of x (diameter) rather than of G, the height of the tree, as in: the height of the tree is changing at the rate of $\frac{100}{(1 + 50)^2} \cdot 0.03 = \frac{3}{2601}$ meter per year.	• When the height is 50 meters, $G(x) = 50 \Rightarrow x = 1.$ Thus, the height of the tree is changing at the rate of $\frac{100}{(1 + 1)^2} \cdot 0.03 = \frac{3}{4}$ meter per year.

Based on your experience at the AP® Reading with student responses, what advice would you offer to teachers to help them improve the student performance on the exam?

Teachers can continue to provide exposure, practice, and feedback to students for contextual problems, including explaining the meaning of variables and of calculated values. Encourage students to review what variables are defined in a problem, and to clearly define any new variables they introduce. Further, encourage and model the use of correct and clear mathematical notation. Find ways to constructively correct flawed notation and clarify how good notation brings clarity and meaning to mathematical expressions.

Teachers can also work on the distinction between approximate and exact values of an expression. For example, in part (a), many responses equated $H'(6)$ to the difference quotient over $[5, 7]$, and some doubled down on the confusion by declaring that $H'(4) = \frac{H(5) - H(3)}{5 - 3} = 2$ in part (b), in effect declaring that $t = 4$ is a value for which $H'(t) = 2$.

Finally, teachers should find ways to encourage student awareness of hypotheses for theorems. Highlight why hypotheses are needed, and explore how theorems may fail in a case where a hypothesis is false. Emphasize that a theorem is more than its "punch line" and that application of a theorem requires verifying that its hypotheses are satisfied.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- The *AP Calculus AB and Calculus BC Course and Exam Description* (*CED*) includes instructional resources for AP Calculus teachers to develop students' broader skills. Please see page 28 of the *CED* for examples of MPACs, questioning, and instructional strategies designed to develop the broader skill of "Reasoning," which was prominent in this question. A table of representative instructional strategies, including definitions and explanations of each, is included on pages 33–37 of the *CED*. The strategy "Construct an argument," for example, may be helpful in developing students' skills at selecting mathematically relevant and accurate data in parts (a) and (c), confirming hypotheses of the Mean Value Theorem in part (b), and providing a logical progression of ideas in setting up the related rates problem in part (d).
- AP Central's course pages for AP Calculus AB and AP Calculus BC include a "Classroom Resources" tab, where you will find a comprehensive resource *Approximation* under the heading "Special Focus Materials." You will also find a variety of teaching modules developed by the College Board over the years. For this question, you might refer to the module *Reasoning from Tabular Data*, which includes commentary, related previous AP Exam questions, student worksheets, teacher notes, and teaching examples.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include "The Exam" tab, where you (and your students) will find questions from previous AP Exams and reflections of chief readers. In addition to end-of-course review, these resources are extremely useful for low-stakes, formative assessment, from which you may base high quality feedback and responsive instruction. Question AB1/BC1 from the 2016 exams and the corresponding chief reader notes would be very helpful in developing the concepts and skills necessary to succeed on question AB4/BC4 on the 2018 exams.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include the "Professional Development" tab, where you will find:
  - Links to Online Modules for Teaching and Assessing AP Calculus. The module *Related Rates* would be relevant to part (d) of this question.
  - A link to Davidson Next, an initiative that, "aims to supplement Advanced Placement (AP) instruction with online modules designed for in-class, blended instruction."
- Finally, the Online Teacher Community is a great place to ask a question, share a strategy, or hear from other AP teachers.

Question AB5 **Topic:** Analysis of Functions/L'Hospital with Graph
Max. Points: 9 **Mean Score:** 2.59

What were the responses to this question expected to demonstrate?

In this problem the function f is defined by $f(x) = e^x \cos x.$ In part (a) students were asked for the average rate of change of f on the interval $0 \le x \le \pi.$ A correct response should demonstrate that the average rate of change is found using a difference quotient, $\frac{f(\pi) - f(0)}{\pi - 0}.$ In part (b) students were asked for the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}.$ A correct response should use the fact that the slope of the tangent line is the value of the derivative of f at the indicated point. The given expression for $f(x)$ can be differentiated using the product rule. In part (c) students were asked to find, with justification, the absolute minimum value of f on the interval $0 \le x \le 2\pi.$ The Extreme Value Theorem guarantees that f attains a minimum on the interval $0 \le x \le 2\pi.$ Candidates for locations of this minimum value are the endpoints of the interval and critical points for f inside the interval. In this case, f is differentiable, so critical points are the two zeros of f', $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}.$ The absolute minimum value of f is the least of the values of f at the four candidates. In part (d) it is given that g is a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0,$ and the graph of the derivative of g for $0 \le x \le 2\pi$ is supplied. Students were asked to find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)},$ if it exists, and to justify their answer. A correct response should start with a confirmation that the requested limit is an indeterminate form to which L'Hospital's Rule applies. The numerator, $f(x),$ has limit 0 as $x \to \frac{\pi}{2},$ using limit properties and continuity of the exponential and cosine functions. Because g is differentiable, it is continuous, so the limit of the denominator, $g(x),$ can be computed by substitution, yielding $\lim_{x \to \pi/2} g(x) = 0.$ After the indeterminate form is confirmed, applying L'Hospital's Rule leads to the limit $\lim_{x \to \pi/2} \frac{f'(x)}{g'(x)},$ which can be evaluated using properties of limits and the provided graph of g'.

For part (a) see LO 2.1A/EK 2.1A1. For part (b) see LO 2.1C/EK 2.1C3, LO 2.3B/EK 2.3B1. For part (c) see LO 1.2B/EK 1.2B1, LO 2.2A/EK 2.2A1. For part (d) see LO 1.1C/EK 1.1C3, LO 2.2B/EK 2.2B2. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating.

How well did the response address the course content related to this question? How well did the responses integrate the skills required on this question?

In part (a), many responses used an appropriate difference quotient to produce the requested average rate of change of f across the interval $[0, \pi].$ In part (b), responses generally showed the connection between the slope of the graph of f and the derivative of f, but many responses had errors in the computation of $f'(x),$ either as a result of an incorrect product rule or incorrect derivatives of the constituent exponential and cosine functions. In part (c), many responses included an effort to find critical points but were often unable to solve for the needed zeros of $f'(x)$ within the interval $[0, 2\pi].$ Responses often lacked identification of the absolute minimum value of f, and presented arguments that fell short of the needed global considerations to justify an absolute minimum. This included giving an incomplete justification for an absolute minimum, either by not considering endpoints or making a relative (local) minimum argument that relied on use of the First Derivative Test or Second Derivative Test only. In part (d), responses generally lacked mention of continuity of g to support the limit of $g(x)$ as x approaches $\frac{\pi}{2}$. In fact, a few responses claimed that g was not continuous at $x = \frac{\pi}{2}$. Although many responses gave an indication of the process for L'Hospital's Rule, the communication of this process was often defeated by poor or no limit notation.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• In part (a), confusing average rate of change with average value, presenting the average rate of change of f across $[0, \pi]$ as $\frac{1}{\pi}\int_0^\pi f(x)\,dx$	• The average rate of change of f across $[0, \pi]$ is $\frac{f(\pi) - f(0)}{\pi - 0}$.
• In part (b), claiming the derivative of a product is the product of derivatives, as in a claim that $f'(x) = e^x \cdot (-\sin x)$	• $f'(x) = e^x \cos x - e^x \sin x$
• In part (c), some responses: ○ gave no or incorrect solutions to $f'(x) = 0$; ○ gave incomplete justification for an absolute minimum, either by not considering endpoints or by making a local argument, such as relying solely on a First Derivative Test or Second Derivative Test; ○ and/or did not identify the absolute minimum value of f.	• $f'(x) = 0 \Rightarrow \cos x = \sin x \Rightarrow x = \frac{\pi}{4},\ x = \frac{5\pi}{4}$ $f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}e^{\pi/4};\ f\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}e^{5\pi/4};$ $f(0) = 1;\ f(2\pi) = e^{2\pi}$ The absolute minimum value of f on $[0, 2\pi]$ is $-\frac{1}{\sqrt{2}}e^{5\pi/4}$.
• In part (d), not exhibiting why evaluation of $g\left(\frac{\pi}{2}\right)$ gives the limit of $g(x)$ as $x \to \frac{\pi}{2}$	• Because g is differentiable, g is continuous, so $\lim\limits_{x\to\pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$.
• In part (d), errors in limit notation and linkage, as in: $\lim\limits_{x\to\pi/2} \frac{f(x)}{g(x)} = \frac{0}{0} = \frac{f'\left(\frac{\pi}{2}\right)}{g'\left(\frac{\pi}{2}\right)} = \frac{-e^{\pi/2}}{2}$	• Because g is differentiable, g is continuous, so $\lim\limits_{x\to\pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$. Also, $\lim\limits_{x\to\pi/2} f(x) = 0$. By L'Hospital's Rule, $\lim\limits_{x\to\pi/2} \frac{f(x)}{g(x)} = \lim\limits_{x\to\pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}$.

Based on your experience at the AP® Reading with student responses, what advice would you offer to teachers to help them improve the student performance on the exam?

Teachers can continue emphasizing careful reading of problems before embarking on a solution. In part (a), hasty reading may have led some students to read "average" and skip over that an average rate of change was requested, not an average value. In part (c), some responses suggested that students who recognized the problem was asking about a minimum, skimmed over that the minimum value of f on $[0, 2\pi]$ was the goal. (This echoes some advice from Question AB1/BC1.)

As mentioned for earlier problems, good use of mathematical notation can be a stumbling point for many responses. In this instance, it was limit notation. Teachers can emphasize where and when limit notation should appear. Also, the symbol $\frac{0}{0}$ is the name of a type of indeterminate form, but not a value; $\frac{0}{0}$ should not be used in an equation where a number would be appropriate.

Finally, teachers can find opportunities for students to practice with the unit circle and values of trigonometric functions at common angles. Like good algebra skills, this precalculus knowledge can be vital to carrying out a calculus procedure.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- The *AP Calculus AB and Calculus BC Course and Exam Description* (*CED*) includes instructional resources for AP Calculus teachers to develop students' broader skills. See page 31 of the *CED* for examples of MPACs, questioning, and instructional strategies designed to develop the broader skill of "Building arguments," which was prominent in this question. A table of representative instructional strategies, including definitions and explanations of each, is included on pages 33–37 of the *CED*. The strategy "Critique reasoning," for example, may be helpful in developing students' skills at accurate application of mathematical terminology in parts (a) and (b), appropriately using a global argument in part (c) to support a conclusion about an absolute minimum, and/or carefully verifying all relevant conditions are met before applying L'Hospital's Rule in part (d).
- Chief Reader Stephen Davis has made available a resource explaining the scoring of questions involving L'Hospital's Rule.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include a "Classroom Resources" tab, where you will find a variety of teaching modules developed by the College Board over the years. For this question, you might refer to the module *Extrema*, which includes commentary, related previous AP exam questions, student worksheets, teacher notes, and teaching examples.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include "The Exam" tab, where you (and your students) will find questions from previous AP Exams and reflections of chief readers. In addition to end-of-course review, these resources are extremely useful for low-stakes, formative assessment, from which you may base high quality feedback and responsive instruction. To find examples of questions involving L'Hospital's Rule you should check the AP Calculus BC exam tab because this topic was only added to the AP Calculus AB exam content in 2017. Question BC5 part (a) from the 2013 exam, question BC4 part (c) from the 2016 exam, and the corresponding chief reader notes would have been helpful in developing the concepts and skills necessary to succeed on question AB5 part (d) on the 2018 exam. At the AP Course Audit website, you will find secure 2017 practice exams illustrating assessment and scoring of L'Hospital's Rule in question AB4/BC4.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include the "Professional Development" tab, where you will find a link to Davidson Next, an initiative that, "aims to supplement Advanced Placement (AP) instruction with online modules designed for in-class, blended instruction." Topics include *L'Hospital's Rule*.
- Finally, the Online Teacher Community is a great place to ask a question, share a strategy, or hear from other AP teachers.

Question AB6

Topic: Separable Differential Equation with Slope Field

Max. Points: 9 **Mean Score:** 3.65

What were the responses to this question expected to demonstrate?

This problem deals with the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$. In part (a) students were given a slope field for the differential equation and asked to sketch solution curves corresponding to solutions that pass through the points $(0, 2)$ and $(1, 0)$. A correct response should be two sketched curves that pass through the indicated points, follow the given slope lines, and extend to the boundaries of the provided slope field. In part (b) students were given that a solution f satisfies $f(1) = 0$ and asked to supply an equation for the line tangent to the graph of f at $x = 1$. Students were then to use this equation to approximate $f(0.7)$. A correct response should use the fact that the slope of the tangent line is the value of the derivative of f at the indicated point, and this value can be computed from substitution of $(x, y) = (1, 0)$ in the differential equation. Combining the slope and the point $(1, 0)$ gives the tangent line equation $y = \frac{4}{3}(x-1)$; substituting $x = 0.7$ into this equation gives the requested approximation for $f(0.7)$. In part (c) students were asked to find the particular solution $y = f(x)$ to the given differential equation that satisfies $f(1) = 0$. A correct response should employ the method of separation of variables and use the initial condition $f(1) = 0$ to resolve the constant of integration to arrive at the solution $f(x) = 2 - \frac{6}{x^2 + 2}$.

For part (a) see LO 2.3F/EK 2.3F1. For part (b) see LO 2.3B/EK 2.3B2. For part (c) see LO 3.3B(a)/EK 3.3B5, LO 3.5A/EK 3.5A2. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating.

How well did the response address the course content related to this question? How well did the responses integrate the skills required on this question?

In part (a), most responses included solution curves through the indicated points and roughly following the given slope lines. In some cases, however, a response curve through the point $(1, 0)$ went out the bottom of the slope field, perhaps indicating that the y-axis is a vertical asymptote, contrary to the slope lines on that axis. In part (b), most responses used the given differential equation to find the required slope and then produced an equation for the line tangent to the graph of f at $(1, 0)$. Some responses, however, opted to solve the initial value problem (i.e., to do part (c) first) and then to find the slope of the tangent line by differentiating the expression found for $f(x)$. In part (c), many responses showed a good effort at separating variables, and most of these responses found valid antiderivatives. Some responses gave an incorrect antiderivative for $\frac{1}{(y-2)^2}$ with respect to y. Some responses made algebra errors in solving for the solution y; this was particularly true for responses that delayed finding the constant of integration until after attempting to solve for y.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• In part (b), some responses attempted to solve the initial value problem for $f(x)$ before finding an equation for the tangent line, instead of finding the slope at $(1, 0)$ from the given differential equation.	• $\left.\frac{dy}{dx}\right\|_{(x,\, y)=(1,\, 0)} = \frac{1}{3} \cdot 1 \cdot (0-2)^2 = \frac{4}{3}$ An equation for the tangent line is $y = \frac{4}{3}(x-1)$.
• In part (c), some responses gave an incorrect antiderivative for $\frac{1}{(y-2)^2}$, moving from $\int \frac{1}{(y-2)^2}\, dy = \int \frac{1}{3}x\, dx$ to $\ln(y-2)^2 = \frac{1}{6}x^2 + C.$	• $\int \frac{1}{(y-2)^2}\, dy = \int \frac{1}{3}x\, dx \Rightarrow \frac{-1}{y-2} = \frac{1}{6}x^2 + C$

Based on your experience at the AP® Reading with student responses, what advice would you offer to teachers to help them improve the student performance on the exam?

For sketching solutions in slope fields, teachers can emphasize that solution curves should extend to the boundary of the given slope field. Such extension isn't sufficiently shown by stopping short of the boundary and drawing arrowheads on the solution curve; arrowheads do not indicate the necessary "bend" of omitted portions of a solution curve. Teachers can encourage students to be aware of what is needed for a problem solution and to take advantage of given information to obtain the needed elements efficiently. In particular, not every tangent line problem need start from a formula for the function to which the line is tangent. Indeed, when proceeding from a differential equation, it may not be possible to solve for a particular solution but relatively easy to find a tangent line approximation to a function value. Also, teachers can continue to find opportunities to reinforce and practice prerequisite skills, such as the algebra used to solve for a particular solution to a separable differential equation.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- The *AP Calculus AB and Calculus BC Course and Exam Description* (*CED*) includes instructional resources for AP Calculus teachers to develop students' broader skills. See page 32 of the *CED* for examples of MPACs, questioning, and instructional strategies designed to develop the broader skill of "Application," which was prominent in this question. A table of representative instructional strategies, including definitions and explanations of each, is included on pages 33–37 of the *CED*. The strategy "Model questions," for example, may be helpful in developing students' skills at constructing graphs using slope fields in part (a), accurately interpreting the notation of the differential equation to find the slope of a line tangent to the graph of a function at a given point in part (b), and finding the particular solution to a differential equation with initial condition in part (c).
- AP Central's course pages for AP Calculus AB and AP Calculus BC include a "Classroom Resources" tab, where you will find an excellent resource *Differential Equations* under the heading "Special Focus Materials." You will also find a variety of articles about differential equations under the heading "From Your AP Colleagues."

- AP Central’s course pages for AP Calculus AB and AP Calculus BC include “The Exam” tab, where you (and your students) will find questions from previous AP Exams and reflections of chief readers. In addition to end-of-course review, these resources are useful for low-stakes, formative assessment, from which you may base high quality feedback and responsive instruction. Virtually all recent exams have examples of free-response questions featuring various aspects of differential equations.
- AP Central’s course pages for AP Calculus AB and AP Calculus BC include the “Professional Development” tab, where you will find a link to Davidson Next, an initiative that, “aims to supplement Advanced Placement (AP) instruction with online modules designed for in-class, blended instruction.” Topics include *Modeling and Solving Differential Equations*.
- Finally, the Online Teacher Community is a great place to ask a question, share a strategy, or hear from other AP teachers.

Question BC2

Topic: Modeling/Improper Integral/Parametric Motion
Max. Points: 9 **Mean Score:** 3.41

What were the responses to this question expected to demonstrate?

The context of this problem is an investigation of plankton cells in a sea. The density of plankton cells at a depth of h meters is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30.$ The density is measured in millions of cells per cubic meter, and the function f is stated to be continuous but is not explicitly given. In part (a) students were asked for the value of $p'(25)$ and to interpret the meaning of $p'(25)$ in the context of the problem. A correct response should give the derivative value as obtained from a graphing calculator and interpret this value as the rate of change of the density of plankton cells, in million cells per cubic meter per meter, at a depth of 25 meters. In part (b) students were asked for the number of plankton cells (to the nearest million) contained in the top 30 meters of a vertical column of water that has horizontal cross sections of constant area 3 square meters. A correct response should combine the density of the plankton, $p(h)$ million cells per cubic meter, and the cross-sectional area of the vertical column to obtain that the number of plankton cells changes at a rate of $3p(h)$ million cells per meter of depth. Thus the number of plankton cells (in millions) in the top 30 meters of the column is the accumulation of this rate for $0 \le h \le 30,$ given by the integral $\int_0^{30} 3p(h)\,dh.$ This integral should be evaluated using a graphing calculator and rounded to the nearest integer. In part (c) a function u is introduced that satisfies $0 \le f(h) \le u(h)$ for $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105.$ Given that the column of water in part (b) is K meters deep, where $K > 30,$ students were asked to write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column, and to explain why the number of plankton cells in the column is at most 2000 million. Using the idea from part (b), a correct response should realize the number of plankton cells in the column is a definite integral of 3 times the density from $h = 0$ to $h = K.$ Because $K > 30,$ and the density is given by $f(h)$ at depths $h \ge 30,$ the number of plankton cells, in millions, in the entire column is $\int_0^{30} 3p(h)\,dh + \int_{30}^{K} 3f(h)\,dh.$ The first term was found in part (b); the second term can be bounded by $3 \cdot 105 = 315$ using the given information about the functions f and u, together with properties of integrals. Summing the answer from part (b) with the upper bound of 315 for the second term shows that the number of plankton cells in the entire column of water is less than 2000 million. In part (d) the position of a research boat on the sea's surface is described parametrically by $(x(t), y(t))$ for $t \ge 0,$ where $x'(t) = 662\sin(5t),$ $y'(t) = 880\cos(6t),$ t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Students were asked to find the total distance traveled by the boat over the time interval $0 \le t \le 1.$

A correct response should find the total distance traveled by the boat as the integral of its speed, $\sqrt{(x'(t))^2 + (y'(t))^2},$ across the time interval $0 \le t \le 1$ and evaluate this integral using a graphing calculator.

For part (a) see LO 2.3A/EK 2.3A1, LO 2.3D/EK 2.3D1. For part (b) see LO 3.3B(b)/EK 3.3B2, LO 3.4E/EK 3.4E1. For part (c) see LO 3.2C/EK 3.2C2, LO 3.2D (BC)/EK 3.2D2 (BC), LO 3.4E/EK 3.4E1. For part (d) see LO 3.3B(b)/EK 3.3B2, LO 3.4C/EK 3.4C2 (BC). This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, building notational fluency, and communicating.

How well did the response address the course content related to this question? How well did the responses integrate the skills required on this question?

In part (a), responses generally showed an accurate evaluation of $p'(25)$ via a graphing calculator. However, responses were off the mark in many ways with regard to interpreting, with correct units, the meaning of this derivative value; failing to give meaning to the 25, identifying p as indicating number of plankton instead of density, not clearly indicating the derivative is a rate of change, and/or missing some aspect of the units for $p'(25)$. In part (b), most responses included an integral whose integrand was a multiple of $p(h)$, but failed to correctly incorporate the cross-sectional area of the column. In part (c), few responses correctly expressed the number of plankton cells using integrals, with many using $u(h)$ instead of $f(h)$. Few responses extended $f(h) \le u(h)$ to $\int_{30}^{K} f(h)\,dh \le \int_{30}^{\infty} u(h)\,dh$. In part (d), responses generally indicated correctly that the total distance traveled by the boat is computed by a definite integral of the boat's speed function. Most issues were a result of errors in presentation or notation.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• In part (a), responses showed errors in all aspects of the meaning of $p'(25)$.	• At a depth of 25 meters, the density of plankton cells is changing at a rate of $p'(25) = -1.179$ million cells per cubic meter per meter.
• In part (b), responses did not include that cross sections of the water column had area 3 square meters, giving the number of plankton cells (in millions) as $\int_0^{30} p(h)\,dh$.	• The number of plankton cells (in millions) in the column is $\int_0^{30} 3p(h)\,dh$.
• In part (c), responses showed confusion between the actual number of plankton cells and a bound on the number of plankton cells.	• The number of plankton cells (in millions) in the entire column of water is given by $\int_0^{30} 3p(h)\,dh + \int_{30}^{K} 3f(h)\,dh = 1675.4149 + \int_{30}^{K} 3f(h)\,dh$. Because $0 \le f(h) \le u(h)$ for $h \ge 30$, $1675.4149 + 3\int_{30}^{K} f(h)\,dh$ $\le 1675.4149 + 3\int_{30}^{\infty} u(h)\,dh$ $= 1675.4149 + 3 \cdot 105 = 1990.4149$. The total number of plankton cells in the column of water is bounded by 1990.4149 million, which is less than 2000 million.
• In part (d), some responses misapplied an arc length formula for a function $y = f(x)$ to this parametric setting, representing the distance traveled by the boat as $\int_0^1 \sqrt{1 + (y'(t))^2}\,dt$.	• The total distance traveled by the boat for $0 \le t \le 1$ is $\int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = 757.456$ meters.

Based on your experience at the AP® Reading with student responses, what advice would you offer to teachers to help them improve the student performance on the exam?

Effective and precise communication continues to be a challenge for students. One aspect of communication is the interpretation of mathematical expressions that pertain to particular contexts, such as the derivative expression in part (a). Teachers can emphasize that every aspect of the derivative expression should be explained with accompanying units.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- The *AP Calculus AB and Calculus BC Course and Exam Description* (*CED*) includes instructional resources for AP Calculus teachers to develop students' broader skills. Please see page 30 of the *CED* for examples of MPACs, questioning, and instructional strategies designed to develop the broader skill of "Interpretation," which was important in this question. A table of representative instructional strategies, including definitions and explanations of each, is included on pages 33–37 of the *CED*. In addition to the strategies provided under this broader skill, the strategy "Error analysis" may be helpful in developing students' skills at identifying what makes a correct and complete interpretation of units and their meaning within the context of the problem.
- AP Central's course page for AP Calculus BC includes a "Classroom Resources" tab, where you will find an excellent teaching module *Vectors* developed by the College Board, which includes resources for reviewing precalculus understandings of parametric equations (see day 1) and teaching parametric equations and calculus (see day 2).
- AP Central's course pages for AP Calculus AB and AP Calculus BC include "The Exam" tab, where you (and your students) will find items from previous AP Exams and reflections of chief readers. In addition to end-of-course review, these resources are extremely useful for low-stakes, formative assessment, from which you may base high-quality feedback and responsive instruction. For example, question BC2 part (d) and its scoring guidelines from the 2015 exam would allow students to practice the skills needed to succeed on question BC2 part (d) on the 2018 exam.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include the "Professional Development" tab, where you will find:
    - Links to Online Modules for Teaching and Assessing AP Calculus. The module *Interpreting Context for Definite Integrals* would be relevant to this question.
    - A link to Davidson Next, an initiative that, "aims to supplement Advanced Placement (AP) instruction with online modules designed for in-class, blended instruction." One of the topics is *Parametric Equations*.
- Finally, the Online Teacher Community is a great place to ask a question, share a strategy, or hear from other AP teachers.

Question BC5

Topic: Polar-Area/Tangent Line/Related Rates
Max. Points: 9 **Mean Score:** 4.23

What were the responses to this question expected to demonstrate?

In this problem a polar graph is provided for polar curves $r = 4$ and $r = 3 + 2\cos\theta$. It was given that the curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$. In part (a) students were asked for an integral expression that gives the area of the region R that is inside the graph of $r = 4$ and outside the graph of $r = 3 + 2\cos\theta$. A correct response should resource the formula for the area of a simple polar region as half of a definite integral of the square of the radius function. The area of R is given by $\frac{1}{2}\int_{\pi/3}^{5\pi/3} 4^2\,d\theta - \frac{1}{2}\int_{\pi/3}^{5\pi/3}(3 + 2\cos\theta)^2\,d\theta = \frac{1}{2}\int_{\pi/3}^{5\pi/3}\left(4^2 - (3 + 2\cos\theta)^2\right)d\theta$. In part (b) students were asked for the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$. A correct response should deal with the conversion between polar and rectangular coordinate systems given by $y = r\sin\theta$ and $x = r\cos\theta$, differentiate these with respect to θ using the product rule, and find the slope of the line tangent to the graph as the value of $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ at $\theta = \frac{\pi}{2}$. In part (c) the motion of a particle along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$ is such that the distance between the particle and the origin increases at a constant rate of 3 units per second. Students were asked for the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$ and to indicate units of measure. A correct response should use the chain rule to relate the rates of r and θ with respect to time t: $\frac{dr}{dt} = -2\sin\theta \cdot \frac{d\theta}{dt}$. Recognizing that $\frac{dr}{dt} = 3$ from the problem statement, it follows that $\left.\frac{d\theta}{dt}\right|_{\theta=\pi/3} = -\sqrt{3}$ radians per second.

For part (a) see LO 3.4D/EK 3.4D1 (BC). For part (b) see LO 2.1C/EK 2.1C7 (BC), LO 2.2A/EK 2.2A4 (BC), LO 2.3B/EK 2.3B1. For part (c) see LO 2.1C/EK 2.1C7 (BC), LO 2.2A/EK 2.2A4 (BC), LO 2.3C/EK 2.3C2. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating.

How well did the response address the course content related to this question? How well did the responses integrate the skills required on this question?

In part (a), some responses treated the region as a difference, in effect a difference of two integrals–a portion of the area inside the circle minus a portion of the area inside the limaçon; other responses combined these into a single integral. Those of the first type often took the area of the entire circle instead of the circular sector from $\theta = \frac{\pi}{3}$ to $\theta = \frac{5\pi}{3}$. Those in the "combined" camp sometimes had an integrand that was a square of a difference instead of the difference of squares. In part (b), many responses expressed x and y in terms of r and θ (or in terms of θ alone) and proceeded with the appropriate calculus process. Some of these, however, were waylaid by algebra errors or sign errors in derivatives of sine and/or cosine functions. A few responses used a value of $\frac{dr}{d\theta}$ for the slope, demonstrating a lack of distinction between rectangular and polar coordinate settings. In part (c), many responses showed attempts at the chain rule to compute $\frac{dr}{dt}$ as $\frac{dr}{d\theta} \cdot \frac{d\theta}{dt}$, although many of these had algebra or trigonometric errors or errors in specifying the requested units.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• In part (a), computing area of the circular sector as the entire area of the circle, as in: $\text{Area} = \pi \cdot 4^2 - \frac{1}{2}\int_{\pi/3}^{5\pi/3} (3 + 2\cos\theta)^2 \, d\theta$	• $\text{Area} = \frac{2}{3} \cdot \pi \cdot 4^2 - \frac{1}{2}\int_{\pi/3}^{5\pi/3} (3 + 2\cos\theta)^2 \, d\theta$
• In part (b), computing the slope of the tangent line as $\frac{dr}{d\theta}$ evaluated at $\theta = \frac{\pi}{2}$	• $\frac{dy}{dx} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\sin\theta)} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$ $\left.\frac{dy}{dx}\right\rvert_{\theta=\pi/2} = \frac{\left(-2\sin\frac{\pi}{2}\right) \cdot \sin\frac{\pi}{2} + 3\cos\frac{\pi}{2}}{\left(-2\sin\frac{\pi}{2}\right) \cdot \cos\frac{\pi}{2} - 3\sin\frac{\pi}{2}} = \frac{-2}{-3} = \frac{2}{3}$
• In parts (b) and (c), many responses incorrectly calculated $\frac{dr}{d\theta} = 2\sin\theta$.	• $\frac{dr}{d\theta} = -2\sin\theta$

Based on your experience at the AP® Reading with student responses, what advice would you offer to teachers to help them improve the student performance on the exam?

Many students struggle to break free of a Cartesian coordinate mindset to enter the polar coordinate framework, and we saw some evidence of this struggle in student responses. However, these struggles were overshadowed by errors in arithmetic, algebra, trigonometry, and the product and chain rules for differentiation. Teachers can find ways to nurture and reinforce prerequisite skills, even as they introduce and explore new topics.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- The *AP Calculus AB and Calculus BC Course and Exam Description* (*CED*) includes instructional resources for AP Calculus teachers to develop students' broader skills. Please see page 29 of the *CED* for examples of MPACs, questioning, and instructional strategies designed to develop the broader skill of "Modeling," which was evident in the area of multiple representations in this question. A table of representative instructional strategies, including definitions and explanations of each, is included on pages 33–37 of the *CED*. The strategies "Create representations" and/or "Marking the text," for example, may be helpful in developing students' skills at extracting key and relevant information at multiple points across the curriculum, including when learning to move between Cartesian and polar coordinates, as in this question. A variation on "Graph and switch," in which students use graphing calculators to explore graphs defined in polar coordinates and then switch graphs and/or calculators to diagnose any errors, might also be helpful.
- AP Central's course page for AP Calculus BC includes a "Classroom Resources" tab, where you will find a variety of teaching modules developed by the College Board over the years. For this question, you might refer to the module *Vectors*. Although this module does not specifically address polar coordinates, resources for teaching parametric equations and calculus presented on day 2 of the module would apply directly to differentiation of the parameterizations of the polar functions defined in this question.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include "The Exam" tab, where you (and your students) will find questions from previous AP exams and reflections of chief readers. In addition to end-of-

course review, these resources are extremely useful for low-stakes, formative assessment, from which you may base high-quality feedback and responsive instruction. Question BC2 from the 2017 exam and the corresponding chief reader notes would have been very helpful in developing the concepts and skills necessary to succeed on question BC5 on the 2018 exam.

- Finally, the Online Teacher Community is a great place to ask a question, share a strategy, or hear from other AP teachers.

Question BC6

Topic: Maclaurin Series/Alternating Series Error Bound
Max. Points: 9 **Mean Score:** 2.65

What were the responses to this question expected to demonstrate?

In this problem the first four nonzero terms and the general term of the Maclaurin series for $\ln(1 + x)$ are given, and the function f is defined by $f(x) = x\ln\left(1 + \frac{x}{3}\right)$. In part (a) students were asked for the first four nonzero terms and the general term of the Maclaurin series for f. A correct response should substitute $\frac{x}{3}$ for x in the supplied terms of the series for $\ln(1 + x)$, multiply the resulting terms by x, and expand so that each term is a constant multiple of a power of x. The general term should also be included. In part (b) students were asked to determine the interval of convergence of the Maclaurin series for f with supporting work for their answer. A correct response should demonstrate the use of the ratio test to determine the radius of convergence of the series and, then, a test of the endpoints of the interval of convergence to determine which endpoints, if any, are to be included in the interval of convergence. In part (c) students were asked to use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$, where $P_4(x)$ is the fourth-degree Taylor polynomial for f about $x = 0$. A correct response should indicate that the alternating series error bound bounds $|P_4(2) - f(2)|$ by the magnitude of the next term in the alternating series formed by evaluating the Taylor series for f about $x = 0$ at $x = 2$.

For part (a) see LO 4.2B/EK 4.2B5. For part (b) see LO 4.1A/EK 4.1A3, LO 4.1A/EK 4.1A6, LO 4.2C/EK 4.2C2. For part (c) see LO 4.2A/EK 4.2A5. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, building notational fluency, and communicating.

How well did the response address the course content related to this question? How well did the responses integrate the skills required on this question?

In part (a), some responses showed attempts at finding the first four nonzero terms via the formula $\frac{f^{(n)}(0)}{n!}x^n$, but these attempts usually were not successful under the demands of algebra and repeated product rules. Some other responses followed the expected route of function composition and multiplication. Again, many of these attempts had algebra errors. In part (b), most responses used the ratio test to determine a radius of convergence and then considered convergence at endpoints for the interval of convergence. However, many responses had poor or no limit notation and/or no use of absolute value in the ratio test for absolute convergence. In part (c), many responses contained no relevant work. Some responses based a bound on the fifth term as opposed to the fifth-degree term.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• In part (a), making algebra errors, such as expanding $x \cdot (-1)^{n+1} \cdot \frac{\left(\frac{x}{3}\right)^n}{n}$ as $(-1)^{n+1} \cdot \frac{x^{n+1}}{3n}$	• $x \cdot (-1)^{n+1} \cdot \frac{\left(\frac{x}{3}\right)^n}{n} = \frac{(-1)^{n+1}}{n \cdot 3^n}x^{n+1}$

• In part (b), many responses announced convergence or divergence at endpoints of the interval of convergence without supporting statements.	• After determining that the radius of convergence is 3: When $x = -3$, the series is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(-3)^{n+1}}{n \cdot 3^n} = \sum_{n=1}^{\infty}\frac{3}{n},$ which diverges by comparison to the harmonic series. When $x = 3$, the series is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3^{n+1}}{n \cdot 3^n} = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{3}{n},$ which converges by the alternating series test. The interval of convergence of the Maclaurin series for f is $-3 < x \le 3$.
• In part (c), some responses used the sixth-degree term as an error bound: $\lvert P_4(2) - f(2)\rvert < \frac{2^6}{5 \cdot 3^5} = \frac{64}{1215}$	• By the alternating series error bound, $\lvert P_4(2) - f(2)\rvert$ is bounded by the magnitude of the next term in the alternating series. $\lvert P_4(2) - f(2)\rvert < \left\lvert -\frac{2^5}{4 \cdot 3^4}\right\rvert = \frac{8}{81}$

Based on your experience at the AP® Reading with student responses, what advice would you offer to teachers to help them improve the student performance on the exam?

Working on new series derived from known series and using the ratio test to determine the radius of convergence for a power series are more places where obtaining a calculus result depends on successful algebraic skills. These are also places to showcase good communication and appropriate use of mathematical notation, including limit notation and absolute value. Teachers can model correct and clear notation and provide constructive feedback to students about their use of notation.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- The *AP Calculus AB and Calculus BC Course and Exam Description* (*CED*) includes instructional resources for AP Calculus teachers to develop students' broader skills. Please see page 32 of the *CED* for examples of MPACs, questioning, and instructional strategies designed to develop the broader skill of "Application." Developing MPAC3, "Implementing algebraic/computational processes," would have benefited many students on this question. A table of representative instructional strategies, including definitions and explanations of each, is included on pages 33–37 of the *CED*. The strategy of "Work backward" may help students to practice refining attempts at the general term of a Maclaurin series by testing whether a general term correctly generates known terms of the series, a useful skill in part (a) of this question.
- AP Central's course page for AP Calculus BC includes a "Classroom Resources" tab, where you will find an excellent, comprehensive resource *Infinite Series* under the heading "Special Focus Materials."

- AP Central's course pages for AP Calculus AB and AP Calculus BC include "The Exam" tab, where you (and your students) will find items from previous AP Exams and reflections of chief readers. In addition to end-of-course review, these resources are extremely useful for low-stakes, formative assessment, from which you may base high-quality feedback and responsive instruction. The series questions (often question BC6) from many previous exams and the corresponding chief reader notes would have been very helpful in developing the concepts and skills necessary to succeed on question BC6 on the 2018 exam.
- AP Central's course pages for AP Calculus AB and AP Calculus BC include the "Professional Development" tab, where you will find a link to Davidson Next, an initiative that, "aims to supplement Advanced Placement (AP) instruction with online modules designed for in-class, blended instruction." Topics include *Series*, *Series Convergence*, and *Series Manipulations*, which are all relevant to this question.
- Finally, the Online Teacher Community is a great place to ask a question, share a strategy, or hear from other AP teachers.