2018

AP CollegeBoard

AP Statistics

Sample Student Responses and Scoring Commentary

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AP® STATISTICS
2018 SCORING GUIDELINES

Question 6

Intent of Question

The primary goals of this question were to assess a student's ability to (1) describe what constitutes a Type II error for a specific hypothesis test; (2) specify a rejection region in terms of values of the sample mean; (3) compute the power of a test for a specific value in the alternative hypothesis; (4) recognize the definition of power; and (5) understand the impact of increasing the sample size on the power of a test.

Solution

Part (a):

A Type II error occurs when the alternative hypothesis is true, but the null hypothesis is not rejected. In this situation a Type II error would happen if the mean systolic blood pressure of the population of employees is greater than 122 mmHg, but the null hypothesis that it is 122 mmHg is not rejected. In other words a Type II error would happen if the mean blood pressure for the population of employees is higher than the national average, but the test does not conclude that it is higher.

Part (b):

The test is one-sided and the standard deviation is known, so the null hypothesis will be rejected if the test statistic $z = \dfrac{\overline{x} - \mu_0}{\dfrac{\sigma}{\sqrt{n}}} > 1.645$. With $\mu = 122,\ \sigma = 15,$ and $n = 100,$ we get $\dfrac{\overline{x} - 122}{1.5} > 1.645$. Therefore, $\overline{x} > 124.4675$.

Part (c):

If the actual population mean is 125, with $\sigma = 15$ and $n = 100,$ then the sampling distribution of $\overline{x}$ is approximately normal with mean of 125 and standard deviation $\dfrac{15}{\sqrt{100}} = 1.5$. Therefore,

$$P(\overline{x} > 124.4675) = P\left(\frac{\overline{x} - \mu_{\overline{x}}}{\sigma_{\overline{x}}} > \frac{124.4675 - 125}{1.5}\right) = P(z > -0.355) = 0.64.$$

Part (d):

The probability found in part (c) is called the power of the test.

Part (e):

If the sample size is increased from 100 to something larger, the probability of rejecting the null hypothesis when the population mean is 125 will be higher than it is for a sample of size 100. Intuitively, more data provide a higher probability of a correct conclusion. The technical explanation is that the rejection region will still be $z > 1.645,$ but the sampling distributions of the sample mean will have a smaller standard deviation; therefore, the minimum value of $\overline{x}$ for which we would reject the null hypothesis would be lower and, in return, the probability the null hypothesis is rejected will increase.

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Question 6 (continued)

Scoring

This question is scored in three sections. Section 1 consists of part (a) and part (b), section 2 consists of part (c), and section 3 consists of part (d) and part (e). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response satisfies the following four components:

1. Part (a) includes in the description of a Type II error the fact that the alternative hypothesis is true, either generically or in context of the situation.
2. Part (a) includes in the description of a Type II error the fact that the null hypothesis is not rejected, either generically or in context of the situation.
3. Part (b) includes a correct *z*-score for the upper 5 percent tail and indicates the correct direction for the rejection region.
4. Part (b) includes $\mu_{\bar{x}} = 122$, $\sigma_{\bar{x}} = 1.5$, and the resulting $\bar{x}$ value.

Partially correct (P) if the response satisfies only two or three of the four components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- If the response in part (a) does not include context, the number of components satisfied is reduced by one (that is, from four to three, or from three to two, and so on). Context includes a reference to units, to blood pressure, to employees, etc.
- If a response in part (a) is clearly referring to an individual's blood pressure as opposed to the mean blood pressure of all employees, neither components 1 nor 2 are satisfied.

Section 2 is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Recognizes that the null hypothesis will be rejected when $\bar{x} \geq 124.4675$, as found in part (b).
2. Provides the correct sampling distribution for the sample mean when the true mean is 125, including correct values for the mean and standard deviation, either explicitly or by plugging them into the test statistic formula.
3. Provides evidence of using the normal curve and finds the correct probability value.

Partially correct (P) if the response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Components 1 and 3 can still be satisfied if errors made in finding the rejection region in part (b) are carried into part (c).
- A calculator statement that does not include labels for input values does not satisfy component 2 but may still satisfy components 1 and 3.

Question 6 (continued)

Section 3 is scored as follows:

Essentially correct (E) if the response satisfies the following four components:

1. Part (d) specifies power as the name of the probability.
2. Part (e) correctly states that the probability would be greater.
3. Part (e) correctly implies that the standard deviation of the sampling distribution decreases, either explicitly or by substituting values into a formula.
4. Part (e) indicates the minimum value of $\bar{x}$ for which the null hypothesis is rejected decreases, either explicitly or by substituting values into a formula.

Note: Component 4 can still be satisfied if a response indicates that a maximum value of $\bar{x}$ for which the null hypothesis is rejected increases if this direction is consistent with answers in parts (b) and (c).

Partially correct (P) if the response satisfies only two or three of the four components.

Incorrect (I) if the response does not meet the criteria for E or P.

Question 6 (continued)

4 Complete Response

Three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 Minimal Response

One section essentially correct

OR

No sections essentially correct and two sections partially correct

OR

Sections 1 and 2 incorrect, and section 3 partially correct with exactly three of the four components satisfied

OR

Section 1 partially correct with exactly three of the four components satisfied, and sections 2 and 3 incorrect

6A1

STATISTICS
SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Systolic blood pressure is the amount of pressure that blood exerts on blood vessels while the heart is beating. The mean systolic blood pressure for people in the United States is reported to be 122 millimeters of mercury (mmHg) with a standard deviation of 15 mmHg.

The wellness department of a large corporation is investigating whether the mean systolic blood pressure of its employees is greater than the reported national mean. A random sample of 100 employees will be selected, the systolic blood pressure of each employee in the sample will be measured, and the sample mean will be calculated.

Let μ represent the mean systolic blood pressure of all employees at the corporation. Consider the following hypotheses.

$$H_0 : \mu = 122$$
$$H_a : \mu > 122$$

(a) Describe a Type II error in the context of the hypothesis test.

Type II error is when you mistakenly fail to reject the H_0 when it is, in reality, false. In this situation, a Type II error would mean that the study concluded that there is insufficient evidence to think that H_0 is false and so the company concludes that the mean population BP of its employees is 122 but in reality the mean population BP of its employees is greater than 122.

(b) Assume that σ, the standard deviation of the systolic blood pressure of all employees at the corporation, is 15 mmHg. If $\mu = 122$, the sampling distribution of $\bar{x}$ for samples of size 100 is approximately normal with a mean of 122 mmHg and a standard deviation of 1.5 mmHg. What values of the sample mean $\bar{x}$ would represent sufficient evidence to reject the null hypothesis at the significance level of $\alpha = 0.05$?

$\mu_{\bar{x}} = 122$

$\sigma_{\bar{x}} = \sigma/\sqrt{n} = \frac{15}{\sqrt{100}} = 1.5$

USE z because σ is known

z of 1.645 or greater would provide p-value < 0.05

$1.645 = \frac{\bar{x} - 122}{\sigma_{\bar{x}}} = \frac{\bar{x} - 122}{1.5}$

$\bar{x} \geq 124.467$ mmHg

The actual mean systolic blood pressure of all employees at the corporation is 125 mmHg, not the hypothesized value of 122 mmHg, and the standard deviation is 15 mmHg.

(c) Using the actual mean of 125 mmHg and the results from part (b), determine the probability that the null hypothesis will be rejected.

From a population of 125 mmHg, the probability of getting a sample with mean 124.467 or higher is:

$\mu = 125$ $\mu_{\bar{x}} = 125$

$\sigma = 15$

$\sigma_{\bar{x}} = \frac{15}{\sqrt{100}} = 1.5$

$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{124.467 - 125}{1.5} = -0.355$

$P(z \geq -0.355) \rightarrow$ via technology 0.6387

Using the actual mean, there is a 63.87% chance that the study in part b will obtain a sample mean of 124.467 mmHg or greater, which will allow the null hypothesis to be rejected.

(d) What statistical term is used for the probability found in part (c) ?

This is the power; the probability that the test will correctly reject a false null hypothesis.

(e) Suppose the size of the sample of employees to be selected is greater than 100. Would the probability of rejecting the null hypothesis be greater than, less than, or equal to the probability calculated in part (c) ? Explain your reasoning.

If n is greater than 100, the power of the test would increase, and thus the probability in part (c) will also increase.

This is because $\sigma_{\bar{x}}$ will decrease, and so the sample mean needed in study (b) to reject the null hypothesis will also decrease. Consequently, the z statistic in part c will decrease (more negative) and the probability of obtaining a sample with the aforementioned mean will increase.

6B1

STATISTICS

SECTION II

Part B

Question 6

Spend about 25 minutes on this part of the exam.

Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Systolic blood pressure is the amount of pressure that blood exerts on blood vessels while the heart is beating. The mean systolic blood pressure for people in the United States is reported to be 122 millimeters of mercury (mmHg) with a standard deviation of 15 mmHg.

The wellness department of a large corporation is investigating whether the mean systolic blood pressure of its employees is greater than the reported national mean. A random sample of 100 employees will be selected, the systolic blood pressure of each employee in the sample will be measured, and the sample mean will be calculated.

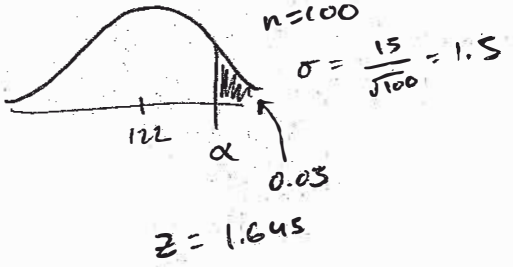
Let μ represent the mean systolic blood pressure of all employees at the corporation. Consider the following hypotheses.

$$H_0 : \mu = 122$$
$$H_a : \mu > 122$$

(a) Describe a Type II error in the context of the hypothesis test.

A type II error would be asserting that the mean systolic blood pressure is 122 mmHg when in reality it is above 122 mmHg

(b) Assume that σ, the standard deviation of the systolic blood pressure of all employees at the corporation, is 15 mmHg. If $\mu = 122$, the sampling distribution of $\bar{x}$ for samples of size 100 is approximately normal with a mean of 122 mmHg and a standard deviation of 1.5 mmHg. What values of the sample mean $\bar{x}$ would represent sufficient evidence to reject the null hypothesis at the significance level of $\alpha = 0.05$?



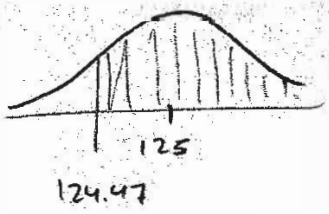
$$\frac{\bar{x} - 122}{1.5} = 1.645$$

124.47 mmHg

The actual mean systolic blood pressure of all employees at the corporation is 125 mmHg, not the hypothesized value of 122 mmHg, and the standard deviation is 15 mmHg.

(c) Using the actual mean of 125 mmHg and the results from part (b), determine the probability that the null hypothesis will be rejected.

$\sigma = \frac{15}{\sqrt{100}} = 1.5$



$$t = \frac{124.47 - 123}{1.5} = -0.3533$$

$P =$ 0.6377

(d) What statistical term is used for the probability found in part (c) ?

Power

(e) Suppose the size of the sample of employees to be selected is greater than 100. Would the probability of rejecting the null hypothesis be greater than, less than, or equal to the probability calculated in part (c) ? Explain your reasoning.

If more employees are sampled, the standard deviation of the distribution would decrease meaning that the $\bar{x}$ needed to reject H_0 goes down. This value is then further from the real 125. This along with the smaller $\sigma_{\bar{x}}$ will make the T-value more negative leading to a larger p-value. The probability would increase.

6C1

STATISTICS
SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Systolic blood pressure is the amount of pressure that blood exerts on blood vessels while the heart is beating. The mean systolic blood pressure for people in the United States is reported to be 122 millimeters of mercury (mmHg) with a standard deviation of 15 mmHg.

The wellness department of a large corporation is investigating whether the mean systolic blood pressure of its employees is greater than the reported national mean. A random sample of 100 employees will be selected, the systolic blood pressure of each employee in the sample will be measured, and the sample mean will be calculated.

Let μ represent the mean systolic blood pressure of all employees at the corporation. Consider the following hypotheses.

$$H_0 : \mu = 122$$
$$H_a : \mu > 122$$

(a) Describe a Type II error in the context of the hypothesis test.

A type II error is incorrectly rejecting the null hypothesis. In this context, a type II error would be concluding that the mean systolic blood pressure of all employees is greater as the reported national mean, even though it is equal to the national mean.

(b) Assume that σ, the standard deviation of the systolic blood pressure of all employees at the corporation, is 15 mmHg. If $\mu = 122$, the sampling distribution of $\bar{x}$ for samples of size 100 is approximately normal with a mean of 122 mmHg and a standard deviation of 1.5 mmHg. What values of the sample mean $\bar{x}$ would represent sufficient evidence to reject the null hypothesis at the significance level of $\alpha = 0.05$?

$\bar{x}$ has to be less than or equal to 119 in order to reject the null hypothesis according to the t distribution

$P(\bar{x} < 122) \leq 0.05 \quad P\left(t < \frac{\bar{x} - \mu_{\bar{x}}}{SD_{\bar{x}}}\right)$

$P\left(t_{99} < \frac{\bar{x} - 122}{1.5}\right) = 0.05$

$\frac{\bar{x} - 122}{1.5} = -1.64485$

$\bar{x} = 119.51$

The actual mean systolic blood pressure of all employees at the corporation is 125 mmHg, not the hypothesized value of 122 mmHg, and the standard deviation is 15 mmHg.

(c) Using the actual mean of 125 mmHg and the results from part (b), determine the probability that the null hypothesis will be rejected.

P

$P\left(t_{99} < \frac{\bar{x} - \mu_{\bar{x}}}{SD_{\bar{x}}}\right)$

$t_{99} < \frac{119 - 125}{15} = t_{99} < -0.4 = 0.345$

34.5% that the null hypothesis will be rejected according to the t distribution.

(d) What statistical term is used for the probability found in part (c) ?

The term is probability of a type II error.

(e) Suppose the size of the sample of employees to be selected is greater than 100. Would the probability of rejecting the null hypothesis be greater than, less than, or equal to the probability calculated in part (c) ? Explain your reasoning.

The probability of rejecting the null hypothesis will be greater because the sample mean needed to reject the null hypothesis would increase. This is due to the $SE_{\bar{x}}$ decreasing as sample size goes up. $\left(\frac{\sigma}{\sqrt{n}}\right)$ This leads to the range of sample means that reject the null hypothesis to increase. When we have a higher sample mean, it gets closer to the mean. This creates a higher probability of rejecting the null hypothesis.

AP® STATISTICS
2018 SCORING COMMENTARY

Question 6

Overview

The primary goals of this question were to assess a student's ability to (1) describe what constitutes a Type II error for a specific hypothesis test; (2) specify a rejection region in terms of values of the sample mean; (3) compute the power of a test for a specific value in the alternative hypothesis; (4) recognize the definition of power; and (5) understand the impact of increasing the sample size on the power of a test.

Sample: 6A
Score: 4

In part (a) the response indicates that a Type II error occurs when the alternative hypothesis is true, the population mean systolic blood pressure is greater than 122, which satisfies component 1 of section 1. The response also indicates the results of the hypothesis test are to fail to reject the null hypothesis, that the population mean blood pressure is equal to 122, which satisfies component 2 of section 1. In part (b) the response recognizes that because the population standard deviation is given, the normal distribution is appropriate for this problem and provides the correct standard normal value for the upper five percent, which satisfies component 3 of section 1. The correct population mean of 122 and standard deviation of the sampling distribution of 1.5 are clearly labeled as the values used in the correct calculations of sample mean values that would reject the null hypothesis, which satisfies component 4 of section 1. Because all four components are satisfied, section 1 was scored as essentially correct. In part (c) the response clearly indicates the probability provided is the probability of getting a sample mean higher than the value found in part (b), which satisfies component 1 section 2. The response provides a clear indication that the population mean used for the calculation is the new value of 125 and that the standard deviation of the sampling distribution is 1.5, which satisfies component 2 of section 2. The response indicates a use of the normal distribution to obtain the correct probability, which satisfies component 3 of section 2. Because all three components are satisfied, section 2 was scored as essentially correct. In part (d) the response correctly indicates that the name of the probability found in part (c) is power, which satisfies component 1 of section 3. In part (e) the response correctly states that the probability found in part (c) will increase if sample size increases, which satisfies component 2 of section 3. The response indicates that increasing the sample size will decrease the standard deviation of the sampling distribution, which satisfies component 3 of section 3. The response also indicates that the value found in part (b) will decrease if the sample size increases, which satisfies component 4 of section 3. Because all four components are satisfied, section 3 was scored as essentially correct. Because three sections were scored as essentially correct, the response earned a score of 4.

Sample: 6B
Score: 3

In part (a) the response indicates that a Type II error occurs when the alternative hypothesis is true, the population mean systolic blood pressure is greater than 122, which satisfies component 1 of section 1. The response also indicates the results of the hypothesis test are to fail to reject the null hypothesis, the population mean blood pressure is equal to 122, which satisfies component 2 of section 1. In part (b) the response correctly indicates the standard normal critical value as 1.645 and uses a drawing to indicate the area that the values would reject the null hypothesis is on the upper tail, which satisfies component 3 of section 1. The response indicates the correct population mean of 122, uses the correct standard deviation of 1.5, and provides the correct value for the minimum value to reject the null hypothesis, which satisfies component 4 of section 1. Because all four components are satisfied, section 1 was scored as essentially correct. In part (c) the response indicates with a drawing that the probability being calculated is the probability greater than the value found in part (b), which satisfies component 1 of section 2. The response indicates the correct population mean of 125 and standard deviation of 1.5, which satisfies component 2 of section 2. The response indicates the use of the *t* distribution, rather than the normal distribution, to calculate the probability

Question 6 (continued)

indicated, which does not satisfy component 3 of section 2. Because only two of the three components are satisfied, section 2 was scored as partially correct. In part (d) the response correctly indicates that the name of the probability found in part (c) is power, which satisfies component 1 of section 3. In part (e) the response correctly states that when the sample size is increased, the probability from part (c) will increase, which satisfies component 2 of section 3. Although the response does not indicate for which distribution the standard deviation decreases in words, the response indicates the standard deviation of the sampling distribution in symbols within the explanation, which satisfies component 3 of section 3. The response does indicate that the minimum value that rejects the null hypothesis, found in part (b), will decrease when sample size increases, which satisfies component 4 of section 3. Because all four components are satisfied, section 3 was scored as essentially correct. Because two sections were scored as essentially correct, and one section was scored as partially correct, the response earned a score of 3.

Sample: 6C
Score: 1

In part (a) the response is describing Type I error rather than Type II error. The response does not indicate that the alternative hypothesis is true, the population mean systolic blood pressure is actually greater than 122, which does not satisfy component 1 of section 1. The response does not indicate that the test fails to reject the null hypothesis that the population mean systolic blood pressure is equal to 122, which does not satisfy component 2 of section 1. In part (b) the response indicates that the critical value for the area for which the null hypothesis is rejected is the lower five percent of the *t* distribution, which does not satisfy component 3 of section 1. The response does indicate the correct population mean of 122, standard deviation of 1.5, and the corresponding sample mean value consistent with the work shown, which satisfies component 4 of section 1. Because only one of four components is satisfied, section 1 was scored as incorrect. In part (c) the response indicates the probability calculated is a probability less than the value found in part (b). The response satisfies component 1 of section 2 because finding the probability in the lower tail is consistent with the response in part (b). The response indicates the correct population mean of 125, but the standard deviation used in the calculations is 15 rather than 1.5, which does not satisfy component 2 of section 2. The response uses a *t* distribution in part (b) and then continues with the *t* distribution in part (c) to find the corresponding probability based on the work provided, which satisfies component 3 of section 2. Because two of the three components are satisfied, section 2 was scored as partially correct. In part (d) the response does not correctly identify the name of the probability found in part (c) as power, which does not satisfy component 1 of section 3. The response indicates that the probability found in part (c) will increase if the sample size increases, which satisfies component 2 of section 3. The response indicates that the standard deviation of the sampling distribution will decrease if sample size is increased, which satisfies component 3 of section 3. The response indicates that the maximum sample mean needed to reject the null hypothesis will increase when the sample size increases because the response has been consistent with finding the lower five percent in the rejection region, and component 3 of section 3 is satisfied. Because three of the four components are satisfied, section 3 was scored as partially correct. Because two sections were scored as partially correct, and one section was scored as incorrect, the response earned a score of 1.