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AP® CollegeBoard

AP® Calculus AB

Sample Student Responses and Scoring Commentary

Inside:

Free Response Question 2

- ☑ Scoring Guideline
- ☑ Student Samples
- ☑ Scoring Commentary

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Question 2

(a) v_P is differentiable $\Rightarrow$ v_P is continuous on $0.3 \le t \le 2.8$.

$$\frac{v_P(2.8) - v_P(0.3)}{2.8 - 0.3} = \frac{55 - 55}{2.5} = 0$$

By the Mean Value Theorem, there is a value c, $0.3 < c < 2.8$, such that $v_P'(c) = 0$.

— OR —

v_P is differentiable $\Rightarrow$ v_P is continuous on $0.3 \le t \le 2.8$.

By the Extreme Value Theorem, v_p has a minimum on $[0.3, 2.8]$.

$v_P(0.3) = 55 > -29 = v_P(1.7)$ and $v_P(1.7) = -29 < 55 = v_P(2.8)$.

Thus v_P has a minimum on the interval $(0.3, 2.8)$.

Because v_P is differentiable, $v_P'(t)$ must equal 0 at this minimum.

$$2 : \begin{cases} 1 : v_P(2.8) - v_P(0.3) = 0 \\ 1 : \text{justification, using} \\ \quad \text{Mean Value Theorem} \end{cases}$$

— OR —

$$2 : \begin{cases} 1 : v_P(0.3) > v_P(1.7) \\ \quad \text{and } v_P(1.7) < v_P(2.8) \\ 1 : \text{justification, using} \\ \quad \text{Extreme Value Theorem} \end{cases}$$

(b)
$$\int_0^{2.8} v_P(t)\,dt \approx 0.3\left(\frac{v_P(0) + v_P(0.3)}{2}\right) + 1.4\left(\frac{v_P(0.3) + v_P(1.7)}{2}\right) + 1.1\left(\frac{v_P(1.7) + v_P(2.8)}{2}\right)$$
$$= 0.3\left(\frac{0 + 55}{2}\right) + 1.4\left(\frac{55 + (-29)}{2}\right) + 1.1\left(\frac{-29 + 55}{2}\right)$$
$$= 40.75$$

1 : answer, using trapezoidal sum

(c) $v_Q(t) = 60 \Rightarrow t = A = 1.866181$ or $t = B = 3.519174$

$v_Q(t) \ge 60$ for $A \le t \le B$

$$\int_A^B \left|v_Q(t)\right|\,dt = 106.108754$$

The distance traveled by particle Q during the interval $A \le t \le B$ is 106.109 (or 106.108) meters.

$$3 : \begin{cases} 1 : \text{interval} \\ 1 : \text{definite integral} \\ 1 : \text{distance} \end{cases}$$

(d) From part (b), the position of particle P at time $t = 2.8$ is

$$x_P(2.8) = \int_0^{2.8} v_P(t)\,dt \approx 40.75.$$

$$x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t)\,dt = -90 + 135.937653 = 45.937653$$

Therefore at time $t = 2.8$, particles P and Q are approximately $45.937653 - 40.75 = 5.188$ (or 5.187) meters apart.

$$3 : \begin{cases} 1 : \int_0^{2.8} v_Q(t)\,dt \\ 1 : \text{position of particle } Q \\ 1 : \text{answer} \end{cases}$$

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2A 1of2

t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	−29	55	48

2. The velocity of a particle, P, moving along the x-axis is given by the differentiable function v_P, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.

(a) Justify why there must be at least one time t, for $0.3 \le t \le 2.8$, at which $v_P'(t)$, the acceleration of particle P, equals 0 meters per hour per hour.

$V_P'(t) = a_P(t)$ $\quad \frac{V_P(2.8) - V_P(0.3)}{2.8 - 0.3} = V_P'(t)$

Since $V_P(t)$ is a continuous and differentiable function, MVT states that there must be at least one time t, for $0.3 \le t \le 2.8$ at which $V_P'(t) = \frac{V_P(2.8) - V_P(0.3)}{2.8 - 0.3} = \frac{55-55}{2.5} = 0.$

(b) Use a trapezoidal sum with the three subintervals [0, 0.3], [0.3, 1.7], and [1.7, 2.8] to approximate the value of $\int_0^{2.8} v_P(t)\,dt$.

$0.3\left(\frac{55+0}{2}\right) + 1.4\left(\frac{-29+55}{2}\right) + 1.1\left(\frac{55+(-29)}{2}\right)$

$8.25 + 18.2 + 14.3$

$\boxed{\int_0^{2.8} V_P(t)\,dt = 40.75}$

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Continue question 2 on page 7.

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(c) A second particle, Q, also moves along the x-axis so that its velocity for $0 \le t \le 4$ is given by $v_Q(t) = 45\sqrt{t}\cos\left(0.063t^2\right)$ meters per hour. Find the time interval during which the velocity of particle Q is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the velocity of particle Q is at least 60 meters per hour.

$$60 = 45\sqrt{t}\cos(0.063t^2)$$

$$t = 1.866, \ 3.519$$

$$\boxed{1.866 \le t \le 3.519}$$

$$\int_{1.866}^{3.519} 45\sqrt{t}\cos(0.063t^2)\,dt = 106.109$$

$$\boxed{106.109 \text{ meters}}$$

(d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.

$$x_P(2.8) = 40.75$$

$$x_Q(2.8) = -90 + \int_0^{2.8} 45\sqrt{t}\cos(0.063t^2)\,dt$$

$$= -90 + 135.938 = 45.938$$

$$45.938 - 40.75 = 5.188$$

$\boxed{\text{The distance between particles } P \text{ and } Q \text{ at } t = 2.8 \text{ is } 5.188 \text{ meters.}}$

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GO ON TO THE NEXT PAGE.

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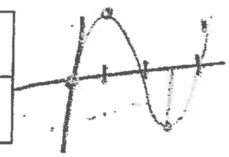
t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	−29	55	48



2. The velocity of a particle, P, moving along the x-axis is given by the differentiable function v_P, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.

(a) Justify why there must be at least one time t, for $0.3 \le t \le 2.8$, at which $v_P'(t)$, the acceleration of particle P, equals 0 meters per hour per hour.

Vp(t) is differentiable and continuous

and $V_P(.3) = 55$ and $V_P(2.8) = 55$ ∴

by Rollels Theorem there exists c value where

$V_P'(c) = 0$

(b) Use a trapezoidal sum with the three subintervals [0, 0.3], [0.3, 1.7], and [1.7, 2.8] to approximate the value of $\int_0^{2.8} v_P(t)\,dt$.

$$\int_0^{2.8} v_P(t)\,dt \approx .3\left(\frac{0+55}{2}\right) + 1.4\left(\frac{84}{2}\right) + 1.1\left(\frac{84}{2}\right)$$

$\approx$ 113.25 meters

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Continue question 2 on page 7.

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2B 2 of 2

(c) A second particle, Q, also moves along the x-axis so that its velocity for $0 \le t \le 4$ is given by $v_Q(t) = 45\sqrt{t}\cos\left(0.063t^2\right)$ meters per hour. Find the time interval during which the velocity of particle Q is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the velocity of particle Q is at least 60 meters per hour.

$V_Q(t) \geq 60$

$t = 1.86618$

X | ✓ at 1.86618

the velocity of particle Q is at least 60 meters per hour on (1.86618, 4] hours.

$\int_{1.86618}^{4} |V_Q(t)|\, dt \approx 132.369$ meters

(d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.

$\int v(t) = s(t)$

$-90 + \int_0^{2.8} V_Q(t) \approx 46.9377$ meters

$\int_0^{2.8} v_P(t)\,dt \approx 113.25$ meters

Particles P and Q are ≈ 67.312 meters apart from each other

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2C 1 of 2

t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	−29	55	48

2. The velocity of a particle, P, moving along the x-axis is given by the differentiable function v_P, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.

(a) Justify why there must be at least one time t, for $0.3 \le t \le 2.8$, at which $v_P'(t)$, the acceleration of particle P, equals 0 meters per hour per hour.

$V_p(t)$ represents the velocity (first derivative) of the particle. From $0.3 \le t \le 2.8$, we see the velocity change signs twice, which represents the graph of $V_p(t)$ moving across the x-axis twice. Therefore by crossing the x-axis (twice), there must be a time where $V'_p(t)$ equals 0.

(b) Use a trapezoidal sum with the three subintervals [0, 0.3], [0.3, 1.7], and [1.7, 2.8] to approximate the value of $\int_0^{2.8} v_P(t)\,dt$.

$$\frac{2.8-0}{2(3)}\left[55+2(26)+26\right] = \boxed{62.067}$$

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Continue question 2 on page 7.

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(c) A second particle, Q, also moves along the x-axis so that its velocity for $0 \le t \le 4$ is given by $v_Q(t) = 45\sqrt{t}\cos\left(0.063t^2\right)$ meters per hour. Find the time interval during which the velocity of particle Q is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the velocity of particle Q is at least 60 meters per hour.

$V_Q(0) = 0$
$V_Q(1) = 44.911$
$V_Q(2) = 61.630$
$V_Q(3) = 65.746$
$V_Q(4) = 48.020$

Velocity of Q is at least 60 m/hour at $(2 \le t \le 3)$

$\int_2^3 |V_Q(t)|\,dt =$ 65.036 m traveled when velocity of Q is at least 60 m/hour

(d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.

At $t = 2.8 \rightarrow P(t)$ is 62.067

$\int_0^{2.8} V_Q(t)\,dt = 135.938 =$ Q position

$\int_0^{2.8} V_P(t)\,dt = 62.067$

$135.938 - 62.067 =$ 73.871 m between P and Q at $t = 2.8$

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Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

In this problem a particle P moves along the x-axis with velocity given by a differentiable function v_P, where $v_P(t)$ is measured in meters per hour and t is measured in hours. The particle starts at the origin at time $t = 0$, and selected values of $v_P(t)$ are given in a table.

In part (a) students were asked to justify why there is at least one time t, for $0.3 \le t \le 2.8$, when the acceleration of particle P is 0. A response should demonstrate that the hypotheses of the Mean Value Theorem are satisfied on the given interval and that applying the Mean Value Theorem to v_P on $[0.3, 2.8]$ leads to the desired conclusion.

In part (b) students were asked to approximate $\int_0^{2.8} v_P(t)\,dt$ using a trapezoidal sum and data from the table of selected values of $v_P(t)$. A response should demonstrate the form of a trapezoidal sum using the three subintervals indicated.

In part (c) a second particle, Q, is introduced, also moving along the x-axis, and with velocity $v_Q(t) = 45\sqrt{t}\cos\left(0.063t^2\right)$ meters per hour. Students were asked to find the time interval during which $v_Q(t) \ge 60$ and to find the distance traveled by particle Q during this time interval. Using a graphing calculator to find the interval, a response should demonstrate that the distance traveled by particle Q is given by the definite integral of the absolute value of v_Q over this time interval. The value of this integral is found using the numerical integration capability of a graphing calculator.

In part (d) students were given that particle Q starts at position $x = -90$ at time $t = 0$ and were asked to use the approximation from part (b) and the velocity function v_Q introduced in part (c) to approximate the distance between particles P and Q at time $t = 2.8$. A response should demonstrate that the integral approximated in part (b) gives the position of particle P at time $t = 2.8$, and that the position of particle Q at this time is found by adding the particle's initial position, $x = -90$, to $\int_0^{2.8} v_Q(t)\,dt$. The student's response should report the difference between these two positions.

For part (a) see LO CHA-2.A/EK CHA-2.A.1, LO FUN-1.B/EK FUN-1.B.1. For part (b) see LO LIM-5.A/EK LIM-5.A.2. For part (c) see LO CHA-4.C/EK CHA-4.C.1, LO LIM-5.A/ EK LIM-5.A.3. For part (d) see LO CHA-4.C/EK CHA-4.C.1, LO LIM-5.A/EK LIM-5.A.3. This problem incorporates all four Mathematical Practices: Practice 1: Implementing Mathematical Processes, Practice 2: Connecting Representations, Practice 3: Justification, and Practice 4: Communication and Notation.

Sample: 2A
Score: 9

The response earned 9 points: 2 points in part (a), 1 point in part (b), 3 points in part (c), and 3 points in part (d). In part (a) the response would have earned the first point in line 3 of the boxed work by presenting the difference

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Question 2 (continued)

quotient $\frac{55-55}{2.5}$. Numerical simplification is not required. The response simplifies the difference quotient to 0 in line 3 of the boxed work, and the first point was earned. In lines 1, 2, and 3 of the boxed work, the response earned the second point with "[s]ince $v_P(t)$ is a continuous and differentiable function, MVT states that there must be at least one time t, for $0.3 \le t \le 2.8$ at which $v_P{}'(t) = \frac{v_P(2.8) - v_P(0.3)}{2.8 - 0.3} = \frac{55-55}{2.5} = 0$." "MVT" is an acceptable form of communication for the Mean Value Theorem. In part (b) the response would have earned the point in line 1 with the trapezoidal sum $0.3\left(\frac{55+0}{2}\right) + 1.4\left(\frac{-29+55}{2}\right) + 1.1\left(\frac{55+(-29)}{2}\right)$. Numerical simplification is not required. The response simplifies the expression to 40.75 in the boxed work and earned the point. In part (c) the response earned the first point in line 3 with the boxed interval $1.866 \le t \le 3.519$. The second point was earned in line 1 on the left with the definite integral $\int_{1.866}^{3.519} 45\sqrt{t}\cos\left(0.063t^2\right)\,dt$. The response earned the third point with the boxed distance 106.109 meters on the right. Units are not required to earn the point. In part (d) the response earned the first point in line 2 with the definite integral $\int_0^{2.8} 45\sqrt{t}\cos\left(0.063t^2\right)\,dt$. The response would have earned the second point in line 3 with the position of particle Q at $t = 2.8$ is $-90 + 135.938$. Numerical simplification is not required. The response simplifies to 45.938 and earned the second point. The response would have earned the third point in line 4 for the difference $45.938 - 40.75$. Numerical simplification is not required. The response simplifies to 5.188, restates this value in the box, and earned the third point. Units are not required to earn the point.

Sample: 2B
Score: 6

The response earned 6 points: 2 points in part (a), no point in part (b), 1 point in part (c), and 3 points in part (d). In part (a) the response earned the first point in line 2 with the equations $v_P(.3) = 55$ and $v_P(2.8) = 55$. The response earned the second point in lines 1, 3, and 4 with "$v_P(t)$ is differentiable and continuous" and "by Rolle's Theorem there exists c value where $v_P{}'(c) = 0$." In part (b) the response did not earn the point because of arithmetic errors in the second and third terms of the trapezoidal sum. This results in an incorrect approximation. In part (c) the response did not earn the first point because of an incorrect interval $(1.86618, 4]$ in line 3 on the right. The response earned the second point in line 4 with the definite integral $\int_{1.86618}^{4} \left|v_Q(t)\right|\,dt$ based on an eligible incorrect interval of $(1.86618, 4]$. The response did not earn the third point because of an incorrect distance. In part (d) the response earned the first point in line 2 with the definite integral $\int_0^{2.8} v_Q(t)$. The missing dt does not impact earning the point. The response earned the second point in line 2 with the position of particle Q at $t = 2.8$ is 45.9377 meters. Units are not required to earn the point. The response earned the third point in line 4 with the position of particle Q at $t = 2.8$ and the imported incorrect value 113.25 from part (b), which are used to compute the consistent answer of 67.312 meters. Units are not required to earn the point.

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Question 2 (continued)

Sample: 2C
Score: 3

The response earned 3 points: no points in part (a), no point in part (b), 1 point in part (c), and 2 points in part (d). In part (a) the response did not earn the first point because there is no correct difference, difference quotient, or comparison of velocities of particle P at times $t = 0.3,\ t = 1.7,$ or $t = 2.8.$ The response is not eligible to earn the second point because the first point was not earned. In part (b) the response did not earn the point because of an incorrect trapezoidal sum. In part (c) the response did not earn the first point because an incorrect interval $2 \le x \le 3$ is presented in line 2 on the right. The response earned the second point in line 6 for the definite integral $\int_2^3 \left|v_Q(t)\right|\,dt$ based on an eligible incomplete interval of $2 \le x \le 3.$ The response did not earn the third point because of an incorrect distance. In part (d) the response earned the first point in line 2 with the definite integral $\int_0^{2.8} v_Q(t)\,dt.$ The response did not earn the second point because the initial condition -90 is not used to determine the position of particle Q at $t = 2.8.$ The response would have earned the third point in line 4 with the identified position of particle Q at $t = 2.8$ and the imported incorrect value 62.067 from part (b), which are used to compute the consistent answer of $135.938 - 62.067.$ Numerical simplification is not required. The response simplifies to 73.871 m and earned the point. Units are not required to earn the point.

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