2019

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Sample Student Responses and Scoring Commentary Set 2

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Free Response Question 1

- ☑ Scoring Guideline
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General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

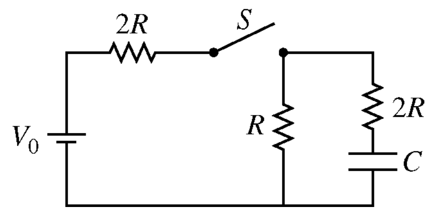
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points



The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(a) LO CNV-7.B.a, SP 5.A, 5.E
2 points

Derive an expression for the steady-state current supplied by the battery.

For using Ohm's law		1 point
$I = \dfrac{V}{R_{eff}} = \dfrac{V_0}{(2R + R)}$		
For correct substitution leading to correct answer		1 point
$I = \dfrac{V_0}{3R}$		

(b) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the charge on the capacitor.

For using the equation relating stored charge to capacitance		1 point
$q = CV = CV_C = CV_R$		
For determining V_R and substituting into the above equation		1 point
$V_R = IR = \left(\dfrac{V_0}{3R}\right)R = \dfrac{1}{3}V_0$		
$q = CV_R = C\left(\dfrac{1}{3}V_0\right) = \dfrac{1}{3}CV_0$		

Question 1 (continued)

(c) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the energy stored in the capacitor.

For any correct equation for energy stored in a capacitor		1 point
$U = \frac{q^2}{2C}$		
Substitute charge and/or voltage from part (b)		1 point
$U = \frac{(CV_0/3)^2}{2C} = \frac{1}{18}CV_0^2$		

Now the switch is opened at time $t = 0$.

(d) LO CNV-7.D.a, SP 5.A, 5.E
2 points

Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

For any correct voltage loop equation		1 point
$V_C - V_R - V_{2R} = 0 \therefore V_C = V_R + V_{2R}$		
$\frac{q(t)}{C} = I(R + 2R)$		
Note: Any correct loop equation for when the switch is open earns the point		
For substituting $-dq/dt$ or dq/dt for the current, consistent with loop equation		1 point
$\frac{q(t)}{C} = -3R\frac{dq}{dt}$		

Question 1 (continued)

(e)

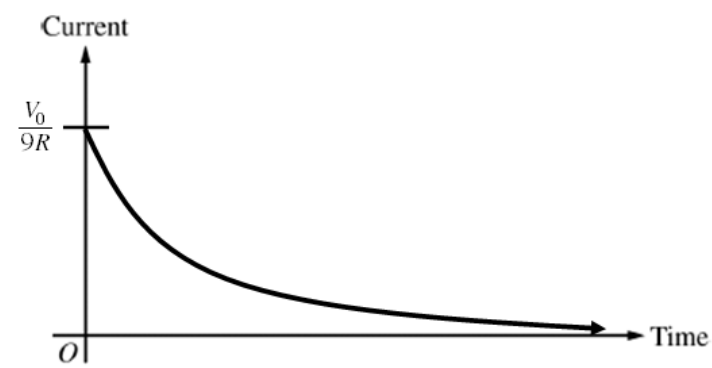
i. LO CNV-7.B.a, SP 5.A, 5.E
2 points

Calculate the current in resistor R immediately after the switch is opened.

For using a voltage consistent with part (b) in $I = \frac{V}{R}$		1 point
For substituting the correct resistance ($3R$) into Ohm's law		1 point
$I = \frac{V_0/3}{3R} = \frac{V_0}{9R}$		

ii. LO CNV-7.E.b, SP 3.C
3 points

On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For a curve that is concave up throughout graph		1 point
For having the horizontal axis as an asymptote		1 point
For labeling the maximum current consistent with part (e)(i)		1 point

Question 1 (continued)

(f) LO CNV-7.G.a, SP 7.A, 7.C
2 points

Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c)?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Equal to"		1 point
For a correct justification invoking conservation of energy		1 point
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Assuming no energy is lost in the wire, then the only parts of the circuit that will dissipate this energy are the two resistors in series with the capacitor.		
Alternate Solution		*Alternate Points*
For selecting "Less than"		*1 point*
For a correct justification invoking conservation of energy		*1 point*
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Additionally there is energy lost due to resistance in the wire and/or energy loss in the capacitor.		

Question 1 (continued)

Learning Objectives

CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-7.B.b: Calculate the stored charge on a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-7.D.a: Derive expressions using calculus to describe the time dependence of the stored charge or potential difference across the capacitor, or the current or potential difference across the resistor in an RC circuit when charging or discharging a capacitor.
CNV-7.E.b: Describe the behavior of the voltage or current behavior over time for a circuit that contains resistors and capacitors in a multi-loop arrangement.
CNV-7.G.a: Describe the energy transfer in charging or discharging a capacitor in an RC circuit.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

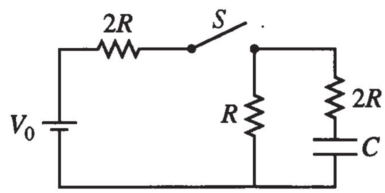
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(a) Derive an expression for the steady-state current supplied by the battery.

cap is fully charged, no current through it

$V_0 = I \cdot (2R + R)$

$\boxed{I = \frac{V_0}{2R}}$ (Amps)

(b) Derive an expression for the charge on the capacitor.

$V_{drop} = I \cdot R = \frac{V_0}{3R} \cdot R = \frac{V_0}{3}$

$V = \frac{Q}{C}$

$\frac{V_0}{3} = \frac{Q}{C}$

$\boxed{Q = \frac{V_0 C}{3}}$ (C)

(c) Derive an expression for the energy stored in the capacitor.

$U = \frac{1}{2} CV^2 = \frac{1}{2} C \left(\frac{Q}{C}\right)^2 = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{\left(\frac{V_0 C}{3}\right)^2}{C}$

$= \frac{1}{2} \frac{V_0^2 C^2}{9C} = \boxed{\frac{1}{18} V_0^2 C}$ (J)

GO ON TO THE NEXT PAGE.

Now the switch is opened at time $t = 0$.

(d) Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

$V_{cap} - I(2R) - I(R) = 0$

$\frac{Q}{C} - 3IR = 0$

$\frac{Q}{C} - 3\left(\frac{dQ}{dt}\right)R = 0 \rightarrow \frac{Q}{C} = 3\frac{dQ}{dt} \cdot R$

$\boxed{\frac{dQ}{dt} \cdot \frac{1}{Q} = \frac{1}{3RC}}$

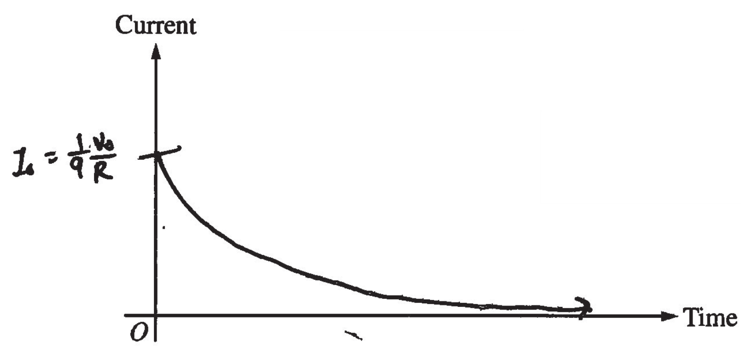
(e)

i. Calculate the current in resistor R immediately after the switch is opened.

$\mathcal{E} = IR$

$I = \frac{\mathcal{E}}{R} = \frac{\left(\frac{V_0}{3}\right)}{3R} = \boxed{\frac{1}{9}\frac{V_0}{R}}$ (A)

ii. On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c) ?

____ Greater than ____ Less than _✓_ Equal to

Justify your answer.

The energy stored in the capacitor leaves the cap. as current, & the resistors dissipate the energy as heat, until, the cap. has no more charge & thus no more current. All the cap.'s was turned into heat by the resistors.

PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
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3 Questions

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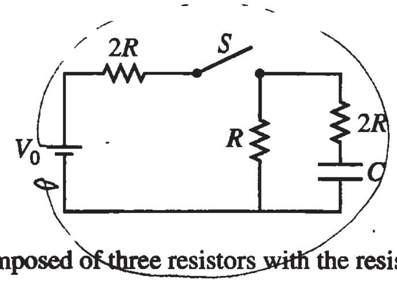


1. The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

   (a) Derive an expression for the steady-state current supplied by the battery.

after close, long time $R_{tot} = 2R + R_? = 2R + R = 3R$

$V = IR$

$V_0 = I \cdot 3R$

$I_s = \frac{V_0}{3R}$

   (b) Derive an expression for the charge on the capacitor.

$V_{capacitor} = V_R$ since they're connected in parallel

$V_R = I_s R = \frac{V_0 R}{3R} = \frac{V_0}{3}$

$V_{capacitor} = \frac{V_0}{3} \Rightarrow Q_{cap} = CV = \frac{CV_0}{3}$

   (c) Derive an expression for the energy stored in the capacitor.

$U_C = \frac{1}{2} Q \Delta V = \frac{1}{2} \cdot \frac{CV_0}{3} \cdot \frac{V_0}{3} = \frac{CV_0^2}{18}$

GO ON TO THE NEXT PAGE.

Now the switch is opened at time $t = 0$.

(d) Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

$V_0 - 2RI - 2R\left(\frac{dQ}{dt}\right) = 0$

Kirchhoff's law.

$-2R\frac{dQ}{dt} = -2RI - V_0$

$$2R\frac{dQ}{dt} = 2RI_{circuit} + V_0$$

(e)

i. Calculate the current in resistor R immediately after the switch is opened.

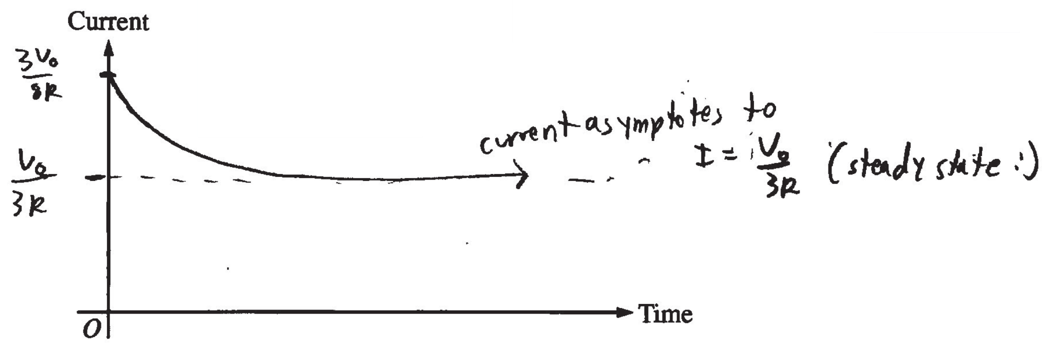
$R_{tot} = 2R + \frac{1}{\frac{1}{R} + \frac{1}{2R}} = 2R + \frac{2}{3}R = \frac{8}{3}R$

$V_0 = IR \rightarrow V_0 = I \cdot \frac{8R}{3} \rightarrow I = \frac{3V_0}{8R}$

current splits proportionally

$I_R = \frac{2}{3} \cdot \frac{3V_0}{8R} = \frac{6V_0}{24R} = \boxed{\frac{V_0}{4R}}$

ii. On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c) ?

X Greater than ____ Less than ____ Equal to

Justify your answer.

Because current never decreases to 0 A, power will continue to be dissipated by the resistors. The total energy dissipated will keep increasing as long as the switch stays open. It will far exceed the calculation in part C.

$P = I^2R$.

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PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

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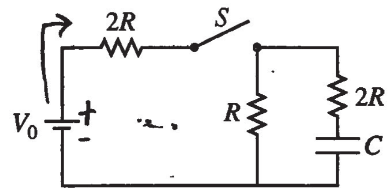

RC circut

1. The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(a) Derive an expression for the steady-state current supplied by the battery.

$V = \varepsilon - IR$ $\quad I = \frac{\Delta V}{R}$ $\quad \Delta V = \frac{1}{4\pi\varepsilon_0}\sum \frac{q_i}{r_i}$ $\quad V = k$

$\Delta V = V_0$ $\quad R = \left(2R + \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1}\right)$ $\quad R_{\parallel} = \sum R \quad R_{series} = \sum \frac{1}{R}$

$I = \frac{2V_0}{7R}$ Ampere (A) $\frac{C}{t}$

(b) Derive an expression for the charge on the capacitor.

$C = \frac{k\varepsilon_0 A}{d}$ Culombs

Q

(c) Derive an expression for the energy stored in the capacitor.

$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$ J

GO ON TO THE NEXT PAGE.

Now the switch is opened at time $t = 0$.

(d) Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

$q(t) = \frac{\varepsilon}{R}\left(1 - e^{R+/C}\right)$

(e)

i. Calculate the current in resistor R immediately after the switch is opened.

$I = V_{0,} - 2R \quad R = \frac{I - 4}{2} \quad \frac{C}{+} \quad A$

ii. On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

(f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c) ?

____ Greater than ____ Less than _X_ Equal to

Justify your answer.

LOCO E

All energy must be conserved ∴ stored in capacitor

GO ON TO THE NEXT PAGE.

Question 1

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses to this question were expected to demonstrate the following:

- How a capacitor acts in a circuit:
  - Current doesn't flow through a completely charged capacitor. In steady-state, the current through the capacitor branch is zero.
  - Where current does flow, $V = IR$.
  - Charge on a capacitor is related to the voltage across it.
  - Energy can be stored in a capacitor.
- How a capacitor discharges:
  - Current and charge are time-dependent during the discharge phase and can be related by a Kirchhoff's loop rule that contains q and dq/dt.
  - The charge on a capacitor decreases exponentially.
  - The current decreases as the charge on the capacitor decreases.
  - The initial current depends on the potential difference across the capacitor and the resistance of the loop.
  - An open switch means current can't flow.
- Energy conservation:
  - Energy in a capacitor can be dissipated in a loop containing resistance.
  - All energy will eventually be dissipated.
  - No energy can be added to a single loop.
- Exponential decrease of current:
  - Curve starts at an initial value (no vertical asymptote).
  - Curve has a zero horizontal asymptote.
  - Rate of decay (slope) decreases.
- Use of a correct original equation to derive a specific result using given symbols.
- Recognizing when current can and cannot flow.
- Graphing exponential decay with proper start and asymptotes.
- Recognizing energy flow in various forms.

Sample: E Q1 A
Score: 14

Part (a) substitutes the correct voltage and resistance into Ohm's law, so 2 points were earned. Part (b) substitutes the correct potential difference into an appropriate equation to calculate charge, so 2 points were earned. Part (c) substitutes the correct charge into an appropriate equation to calculate stored energy, so 2 points were earned. Part (d) substitutes the correct resistance into a correct differential equation but has an incorrect sign on the dq/dt term, so 1 point was earned. Part (e)(i) substitutes the correct voltage and resistance into Ohm's law, so 2 points were earned. Part (e)(ii) has a concave up curve, indicates that the horizontal axis is an asymptote, and correctly labels the maximum value, so 3 points were earned. Part (f) has correct selection and justification, so 2 points were earned.

Question 1 (continued)

Sample: E Q1 B
Score: 7

Parts (a), (b), and (c) earned full credit, 2 points each. Part (d) has an incorrect differential equation and an incorrect sign on the dq/dt term, so no points were earned. Part (e)(i) does not use the voltage from part (b) and uses an incorrect resistance in Ohm's law, so no points were earned. Part (e)(ii) has a concave up curve, but the horizontal axis is not an asymptote, and the label for maximum value is incorrect, so 1 point was earned. Part (f) has an incorrect selection and justification, so no points were earned.

Sample: E Q1 C
Score: 3

Part (a) uses Ohm's law but has an incorrect resistance, so 1 point was earned. Part (b) does not use an appropriate equation to calculate charge, so no points were earned. Part (c) does not use an appropriate equation to calculate stored energy, so no points were earned. Part (d) does not use a differential equation, so no points were earned. Part (e)(i) does not use the voltage from part (b) and uses an incorrect resistance in Ohm's law, so no points were earned. Part (e)(ii) has a concave down graph with no horizontal asymptote, and the maximum voltage is not consistent with part (e)(i), so no points were earned. Part (f) has correct selection and justification, so 2 points were earned.