
AP[®] Statistics

Sample Student Responses and Scoring Commentary

Inside:

Free Response Question 3

- Scoring Guideline**
- Student Samples**
- Scoring Commentary**

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2019 SCORING GUIDELINES

Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) use information from a two-way table of relative frequencies to compute joint, marginal, and conditional probabilities; (2) recognize whether two events are independent; and (3) compute a probability for a binomial distribution.

Solution

Part (a):

(i) $P(\text{never and woman}) = 0.0636$

(ii)

$$\begin{aligned} P(\text{never or woman}) &= P(\text{never}) + P(\text{woman}) - P(\text{never and woman}) \\ &= 0.12 + 0.53 - 0.0636 \\ &= 0.5864 \end{aligned}$$

(iii) $P(\text{never} \mid \text{woman}) = \frac{P(\text{never and woman})}{P(\text{woman})} = \frac{0.0636}{0.53} = 0.12$

Part (b):

Yes, the event of being a person who responds never is independent of the event of being a woman because

$$P(\text{never} \mid \text{woman}) = P(\text{never}) = 0.12.$$

Part (c):

Define X as the number of people in a random sample of five people who always take their medicine as prescribed. Then X has a binomial distribution with $n = 5$ and $p = 0.54$, and

$$P(X \geq 4) = \binom{5}{4}(0.54)^4(0.46)^1 + \binom{5}{5}(0.54)^5(0.46)^0 \approx 0.19557 + 0.04592 \approx 0.24149.$$

Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response reports correct values of the probabilities for (i), (ii), and (iii).

Partially correct (P) if only one or two of the probabilities are correct.

Incorrect (I) if none of the probabilities are correct.

Notes:

- Assuming independence for events never and woman in (i) without referencing the result in part (b) does not satisfy (i).
- Alternative solutions for (ii) include $0.0564 + 0.0636 + 0.1384 + 0.3280 = 0.5864$ and $0.0564 + 0.53 = 0.5864$.

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2019 SCORING GUIDELINES

Question 3 (continued)

Part (b) is scored as follows:

Essentially correct (E) if the response indicates that the events are independent, gives an explanation of independence using the events in the problem, *AND* provides appropriate justification using numbers from the table.

Note: Examples of valid explanations with appropriate justifications include:

- $P(\text{woman and never}) = 0.0636$ is the same as $P(\text{woman}) \times P(\text{never}) = (0.53)(0.12) = 0.0636$.
- $P(\text{never} \mid \text{woman}) = \frac{0.0636}{0.53} = 0.12$ is the same as $P(\text{never}) = 0.12$.
- $P(\text{woman} \mid \text{never}) = \frac{0.0636}{0.12} = 0.53$ is the same as $P(\text{woman}) = 0.53$.

Partially correct (P) if the response indicates that the events are independent *AND* gives an explanation of independence using the events in the problem but does not provide justification using numbers from the table

OR

if the response uses a correct method of illustrating that events are independent but makes an arithmetic mistake or a transcription mistake that results in concluding that these two events are not independent.

Incorrect (I) if the response does not satisfy requirements for E or P.

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Clearly indicates a binomial distribution with $n = 5$ and $p = 0.54$.
2. Indicates the correct boundary value and direction of the event.
3. Reports the correct probability.

Partially correct (P) if the response satisfies component 1 but it does not satisfy one or both of the other two components

OR

if the response does not satisfy component 1 but both of the other two components are satisfied.

Incorrect (I) if the response does not meet the criteria for E or P.

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Question 3 (continued)

Notes:

- The response $B(5, 0.54)$ satisfies component 1.
- Components 1 and 2 are satisfied by displaying the correct formula for computing the binomial probability using the correct values for n and p , e.g.,

$$\binom{5}{4}(0.54)^4(0.46)^1 + \binom{5}{5}(0.54)^5(0.46)^0$$

- Only component 1 is satisfied if the correct binomial distribution is used in an incorrect probability formula, e.g.,

$$\binom{5}{4}(0.54)^4(0.46)^1$$

- For component 2, the boundary value and direction may be described in words, e.g., $P(\text{at least four people})$.
- Component 2 may be satisfied by displaying a bar graph of a binomial distribution with the appropriate bars shaded.
- The response of $1 - \text{binomcdf}(n = 5, p = 0.54, \text{upper bound} = 3) = 0.24$ is scored E since n , p , and the boundary value are clearly identified.

The response of $1 - \text{binomcdf}(n = 5, p = 0.54, 3) = 0.24$ is scored P since n and p are clearly identified and the boundary value is not identified.

The response of $1 - \text{binomcdf}(5, 0.54, 3) = 0.24$ is scored I.

- A normal approximation to the binomial is not appropriate since $np = 5 \times 0.54 = 2.7$ and $2.7 < 5$. A response using the normal approximation can score at most P. To score P, the response must include all of the following:

- an indication that the probability calculated is a normal approximation for the binomial probability
- a correct mean and standard deviation based on the binomial parameters
- clear indication of boundary and direction with a z-score or diagram
- the probability computed correctly.

An example of a response which meets these four criteria is

$$P\left(Z \geq \frac{4 - np}{\sqrt{np(1 - p)}}\right) = P\left(Z \geq \frac{4 - (5)(0.54)}{\sqrt{(5)(0.54)(0.46)}}\right) \approx 0.1217$$
, and the binomial distribution is mentioned.

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2019 SCORING GUIDELINES

Question 3 (continued)

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

3A1

3A1

3. A medical researcher surveyed a large group of men and women about whether they take medicine as prescribed. The responses were categorized as never, sometimes, or always. The relative frequency of each category is shown in the table.

	Never	Sometimes	Always	Total
Men	0.0564	0.2016	0.2120	0.4700
Women	0.0636	0.1384	0.3280	0.5300
Total	0.1200	0.3400	0.5400	1.0000

- (a) One person from those surveyed will be selected at random.

- (i) What is the probability that the person selected will be someone whose response is never and who is a woman?

$$P(\text{Never} \cap \text{Woman}) = 0.0636$$

There is a 0.0636 probability the person selected will be someone whose response is never and who is a woman.

- (ii) What is the probability that the person selected will be someone whose response is never or who is a woman?

$$P(\text{Never} \cup \text{Woman}) = P(\text{Never}) + P(\text{Woman}) - P(\text{Never} \cap \text{Woman}) = 0.12 + 0.53 - 0.0636 = 0.5864$$

The probability that the person selected will be someone whose response is never or who is a woman is 0.5864.

- (iii) What is the probability that the person selected will be someone whose response is never given that the person is a woman?

$$P(\text{Never} | \text{Woman}) = \frac{P(\text{Never} \cap \text{Woman})}{P(\text{Woman})} = \frac{0.0636}{0.53} = 0.12$$

The probability that the person selected will be someone whose response is never given that the person is a woman is 0.12.

- (b) For the people surveyed, are the events of being a person whose response is never and being a woman independent? Justify your answer.

Independence: $P(A) = P(A|B)$
 $P(\text{never}) = P(\text{never} | \text{woman})$
 $0.12 = 0.12$

Yes, these events are independent. Because the probability of selecting a person who responds never, 0.12, is the same as the probability of selecting someone whose response is never given that the person is a woman (also 0.12), the probability of selecting a person whose response is never is not affected by the probability of selecting a woman from the people surveyed, and therefore, the events of being a person who responds never and being a woman are independent.

3A2

3A2

(c) Assume that, in a large population, the probability that a person will always take medicine as prescribed is 0.54. If 5 people are selected at random from the population, what is the probability that at least 4 of the people selected will always take medicine as prescribed? Support your answer.

$X =$ # of people selected out of 5 that always take medicine as prescribed

- i. There are 2 outcomes: success (always) or failure (not always).
- ii. probability of success remains the same for each person at 54%.
- iii. each person's response is independent.

X is a Bernoulli random variable because the above conditions are met. Because measures the number of successes in a finite amount of trials, X is a binomial random variable with parameters $(5, 0.54)$.

$$\begin{aligned} P(X \geq 4) &= 1 - P(X \leq 3) \\ &= 1 - \text{binomcdf}(n, p, x) \\ &= 1 - \text{binomcdf}(5, 0.54, 3) \\ &= 0.241 \end{aligned}$$

According to the binomial model, there is a 24.1% chance at least 4 of 5 people selected will always take medicine as described.

301

301

3. A medical researcher surveyed a large group of men and women about whether they take medicine as prescribed. The responses were categorized as never, sometimes, or always. The relative frequency of each category is shown in the table.

	Never	Sometimes	Always	Total
Men	0.0564	0.2016	0.2120	0.4700
Women	0.0636	0.1384	0.3280	0.5300
Total	0.1200	0.3400	0.5400	1.0000

- (a) One person from those surveyed will be selected at random.

- (i) What is the probability that the person selected will be someone whose response is never and who is a woman?

$$P(\text{never} \cap \text{woman}) = 0.0636$$

The probability that someone who's response is never and is a woman is 0.0636

- (ii) What is the probability that the person selected will be someone whose response is never or who is a woman?

$$P(\text{never} \cup \text{woman}) = P(\text{never}) + P(\text{woman}) - P(\text{never} \cap \text{woman}) = 0.12 + 0.53 - 0.0636 = 0.5864$$

The probability that you pick someone who said never or is a woman is 0.5864

- (iii) What is the probability that the person selected will be someone whose response is never given that the person is a woman?

$$P(\text{never} | \text{woman}) = \frac{P(\text{never} \cap \text{woman})}{P(\text{woman})} = \frac{0.0636}{0.53} = 0.12$$

The probability that you pick someone who said never given that they were a woman is 0.12.

- (b) For the people surveyed, are the events of being a person whose response is never and being a woman independent? Justify your answer.

Yes they are independent since the probability of someone saying never given that they are a woman and the probability of someone saying never should be the same in order for the two events to be independent. In this case both probabilities equal 0.12, hence the events of saying never and being a woman are independent.

3B2

3B2

- (c) Assume that, in a large population, the probability that a person will always take medicine as prescribed is 0.54. If 5 people are selected at random from the population, what is the probability that at least 4 of the people selected will always take medicine as prescribed? Support your answer.

$$P(\text{at least 4 people take their medicine}) = 1 - P(\text{up to 3 people take their medicine}) = 1 - \text{Binompdf}(5, 0.54, 3) = 1 - 0.759 = 0.241$$

The probability of at least 4 people taking their medicine as prescribed is 0.241.

3C1

3C1

3. A medical researcher surveyed a large group of men and women about whether they take medicine as prescribed. The responses were categorized as never, sometimes, or always. The relative frequency of each category is shown in the table.

	Never	Sometimes	Always	Total
Men	0.0564	0.2016	0.2120	0.4700
Women	0.0636	0.1384	0.3280	0.5300
Total	0.1200	0.3400	0.5400	1.0000

- (a) One person from those surveyed will be selected at random.

- (i) What is the probability that the person selected will be someone whose response is never and who is a woman?

$$P(\text{Never} \cap \text{woman}) =$$

$$.12 \times .53 = \boxed{.0636}$$

- (ii) What is the probability that the person selected will be someone whose response is never or who is a woman?

$$P(\text{Never} \cup \text{woman}) =$$

$$.12 + .53 - .0636 = \boxed{.5864}$$

- (iii) What is the probability that the person selected will be someone whose response is never given that the person is a woman?

$$P(\text{Never} | \text{woman}) = \frac{.0636}{.53} = \boxed{.12}$$

- (b) For the people surveyed, are the events of being a person whose response is never and being a woman independent? Justify your answer.

$$P(\text{Never}) = P(\text{Never} | \text{woman})$$

$$.12 = .12 \quad \checkmark$$

302

302

- (c) Assume that, in a large population, the probability that a person will always take medicine as prescribed is 0.54. If 5 people are selected at random from the population, what is the probability that at least 4 of the people selected will always take medicine as prescribed? Support your answer.

$$\text{binomcdf}(5, .54, 4, 00) = \boxed{.241}$$
$$\frac{5}{4} (.54)^4 (.46)^1 + (.54)^5 (.46)^0$$

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GO ON TO THE NEXT PAGE.

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2019 SCORING COMMENTARY

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The primary goals of this question were to assess a student's ability to (1) use information in a two-way table of relative frequencies to compute joint, marginal, and conditional probabilities; (2) recognize if events are independent; and (3) compute a probability for a binomial distribution.

This question primarily assesses skills in skill category 3: Using Probability and Simulation. Skills required for responding to this question include (3.A) determine relative frequencies, proportions or probabilities using simulation or calculations and (3.B) determine parameters for probability distributions.

This question covers content from Unit 4: Probability Rules, Random Variables, and Probability Distributions of the course framework in the AP Statistics Course and Exam Description. Refer to topics 4.3, 4.5, 4.6, and 4.10, and learning objectives VAR-4.A, VAR-4.D, VAR-4.E, UNC-3.B, and UNC-3.C.

Sample: 3A

Score: 4

In part (a)(i) the response correctly identifies the joint probability given in the two-way table. In part (a)(ii) the formula for the union of events is used correctly. In part (a)(iii) correct events are identified and used in the conditional probability formula. The fraction with correct numerator and denominator is given and the resulting probability is correct. Because the correct values of the probabilities are given for (i), (ii), and (iii), part (a) was scored as essentially correct.

In part (b) the correct decision is given. An explanation of independence uses events from the problem, " $P(\text{never}) = P(\text{never} \mid \text{woman})$," with numbers from the table used in the justification. Although not required, the paragraph supports the numerical calculation. Part (b) was scored as essentially correct.

In part (c) the statement, " $1 - \text{binomcdf}(5, 0.54, 3)$," with the parameters identified in the line above it, satisfies components 1 and 2. Component 2 is also satisfied with " $P(X \geq 4)$." The correct probability is given; therefore, component 3 is satisfied. While the binomial distribution is correctly justified, this justification is not required. Because all three components were satisfied, part (c) was scored as essentially correct.

Because three parts were scored as essentially correct, the response earned a score of 4.

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Question 3 (continued)

Sample: 3B

Score: 3

In part (a)(i) the response correctly identifies the joint probability given in the two-way table. In part (a)(ii) the formula for the union of events is correctly used. In part (a)(iii) correct events are identified and used in the conditional probability formula. The fraction with correct numerator and denominator is given, and the resulting probability is correct. Because the correct values of the probabilities are given for (i), (ii), and (iii), part (a) was scored as essentially correct.

In part (b) a valid explanation is given for the decision. That explanation is justified by “both probabilities equal 0.12.” Part (b) was scored as essentially correct.

In part (c) the statement “ $1 - \text{Binomcdf}(5, 0.54, 3)$ ” does not clearly identify the parameters of the binomial, so component 1 is not satisfied. Component 2 is satisfied with the statement “P(at least 4 people take their medicine).” That component is also satisfied by the statement “ $1 - P(\text{up to 3 people take their medicine})$.” The correct probability is given; therefore, component 3 is satisfied. Because component 1 is not satisfied and components 2 and 3 are satisfied, part (c) was scored as partially correct.

Because two parts were scored as essentially correct and one part was scored as partially correct, the response earned a score of 3.

Sample: 3C

Score: 2

In part (a)(i) independence is assumed, and the joint probability is found by multiplying the two marginal probabilities. In part (a)(ii) the formula for the union of events is used correctly. In part (a)(iii) correct events are identified and used in the conditional probability formula. The fraction with correct numerator and denominator is given, and the resulting probability is correct. Because the correct values of the probabilities for (ii) and (iii) are given and independence is assumed in (i) without referencing the result in part (b), part (a) was scored as partially correct.

In part (b) the decision is not communicated. The response has a valid explanation and justification of independence using the correct numbers from the problem. If the response had included “yes, the events are independent,” this part would have been scored essentially correct. Part (b) was scored as incorrect.

In part (c) component 1 is satisfied. The binomial distribution and parameters are identified by using the formula. Component 2 is satisfied with the correct two terms of the binomial distribution formula. The correct probability is given; therefore, component 3 is satisfied. Because all three components are satisfied, part (c) was scored as essentially correct.

Because one part was scored as essentially correct, one part was scored as partially correct, and one part was scored as incorrect, the response earned a score of 2.