2019

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Scoring Guidelines
Set 1

AP® PHYSICS
2019 SCORING GUIDELINES

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

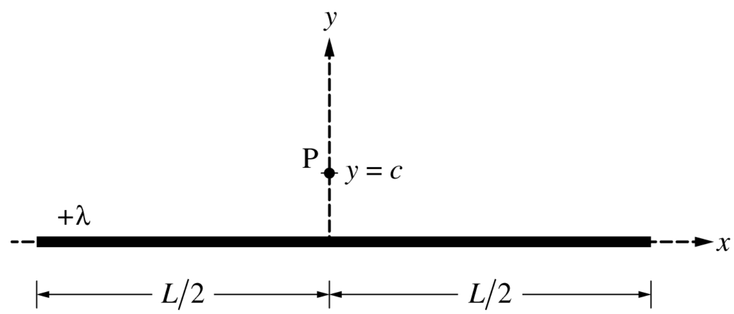
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points



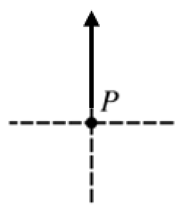
Note: Figure not drawn to scale.

A very long, thin, nonconducting cylinder of length L is centered on the y-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the y-axis at $y = c$, where $L >> c$.

(a)

i. LO CNV-3.B.a, SP 7.A
1 point

On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.



For drawing an arrow at point P that points upward		1 point

ii. LO CNV-2.F, SP 7.C
1 point

Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.

For describing a Gaussian surface that could be used to determine the electric field at point P		1 point
Example: Drawing a cylinder that is coaxial with the thin cylinder and whose surface contains point P can be used to determine the electric field at point P.		
Note: Credit is earned if the student draws the correct surface on the figure.		

Question 1 (continued)

(a) continued

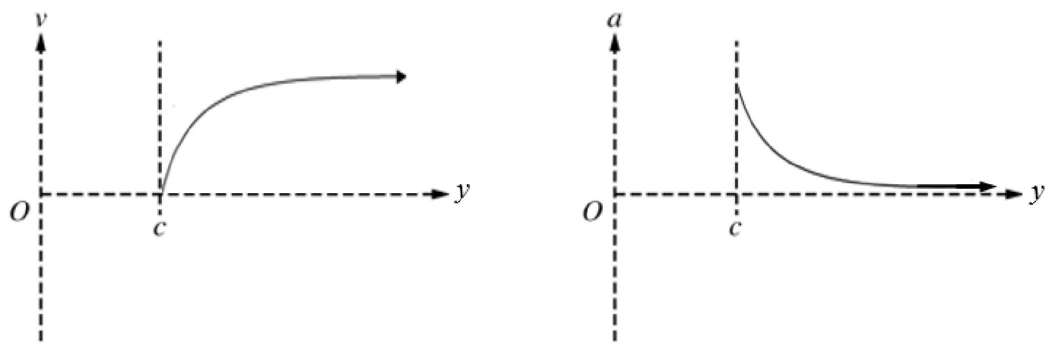
iii. LO CNV-2.C, SP 5.A, 5.E
3 points

Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of λ, c, L, and physical constants, as appropriate.

For using Gauss's law to determine the electric field at point P		1 point
$\frac{q_{enc}}{\varepsilon_0} = \int E \bullet dA \therefore \frac{Q}{\varepsilon_0} = EA$		
For correctly substituting for the charge into the equation above		1 point
For correctly substituting for the area or into the equation above		1 point
$\frac{\lambda L}{\varepsilon_0} = E(2\pi cL) \therefore E = \frac{\lambda}{2\pi\varepsilon_0 c} = \frac{2k\lambda}{c}$		

(b) LO ACT-1.D, SP 3.C
2 points

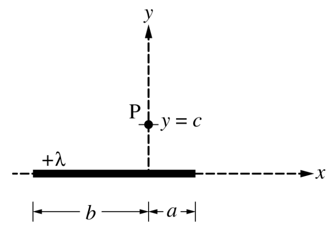
A proton is released from rest at point P. On the axes below, sketch the velocity v as a function of <u>position</u> y and the acceleration a as a function of <u>position</u> y for the proton.



For a concave down graph for v as a function of position x that does not start at the origin		1 point
For a concave up graph for a as a function of position x that has an asymptote at the horizontal axis		1 point

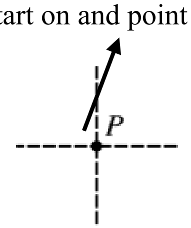
Question 1 (continued)

The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the y-axis is a, and the length of the cylinder to the left of the y-axis is b, where $a < b$.



(c) LO CNV-3.B.a, SP 7.A
2 points

On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.



For drawing an arrow at point P that points to the right		1 point
For drawing an arrow at point P that points up and to the right		1 point

(d)
i. and ii. LO CNV-2.F, SP 7.A
1 point

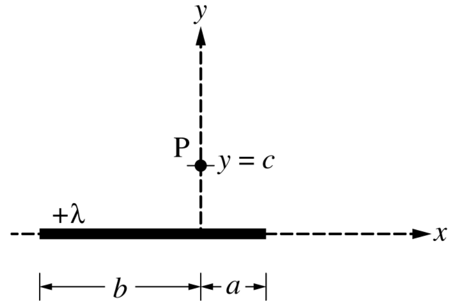
Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?

____ Yes ____ No

If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.

For selecting "No" with a valid explanation		1 point
Claim: Select "No." Evidence: The length of the cylinder is not much greater than the distance from the cylinder to point P and the charge distribution is asymmetric. Reasoning: Therefore, cannot use the approximation of the constant magnitude of electric field over a cylindrical surface.		

Question 1 (continued)



Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

$$E = \int \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2}\, dq$$

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

Horizontal component: $$\left|E_x\right| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{x}{\left(c^2 + x^2\right)^{3/2}}\, dx$$

Vertical component: $$\left|E_y\right| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{y}{\left(c^2 + x^2\right)}\, dy$$

(e)

i. LO CNV-3.A, SP 7.A
1 point

One of the two expressions above is not correct. Which expression is not correct?

____ Horizontal component ____ Vertical component

For correctly selecting “Vertical component”		1 point

Question 1 (continued)

(e) continued
 ii. LO CNV-3.A, SP 7.D
 4 points

Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

For indicating the integral is not along the length of the cylinder		1 point
For an appropriate correction		1 point
Claim: Change *dy* to *dx*. Evidence: The integral is not along the cylinder. Reasoning: This change will make the integral valid.		
For indicating the power on the denominator term for the vertical component is incorrect		1 point
For an appropriate correction		1 point
Claim: The power on the term in the denominator should be $3/2$. Evidence: The units of the integrand are not valid. Reasoning: This change will make the integrand valid.		

Learning Objectives

ACT-1.D: Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.
CNV-2.C: State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.
CNV-2.F: Describe the general features of an unknown charge distribution, given other features of the system.
CNV-3.A: Derive expressions for the electric field of specified charge distributions using integration and the principle of superposition. Examples of such charge distributions include a uniformly charged wire, a thin ring of charge (along the axis of the ring), and a semicircular or part of a semicircular arc.
CNV-3.B.a: Identify and qualitatively describe situations in which the direction and magnitude of the electric field can be deduced from symmetry considerations and understanding the general behavior of certain charge distributions.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.
7.D: Provide reasoning to justify a claim using physical principles or laws.

Question 2

15 points

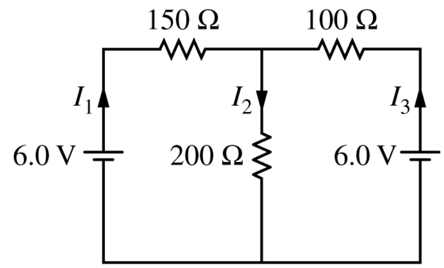


Figure 1

The circuit shown above is constructed with two 6.0 V batteries and three resistors with the values shown. The currents I_1, I_2, and I_3 in each branch of the circuit are indicated.

(a)

i. LO CNV-6.F.b, SP 6.A
3 points

Using Kirchhoff's rules, write, but DO NOT SOLVE, equations that can be used to solve for the current in each resistor.

For an equation representing the sum of the currents at one of the junctions		1 point
$I_1 - I_2 + I_3 = 0$		
For an equation representing the sum of the potential differences around one of the loops		1 point
For an equation representing the sum of the potential differences around a loop different from the above loop		1 point
$6 - 150I_1 - 200I_2 = 0$		
$6 - 100I_3 - 200I_2 = 0$		
$6 - 150I_1 + 100I_3 - 6 = 0$		
Note: Full credit is earned for two correct loop equations using loop currents.		

ii. LO CNV-6.F.b, SP 6.C
2 points

Calculate the current in the 200 Ω resistor.

For combining the equations from part (a)(i)		1 point
$I_1 - I_2 + I_3 = 0$ $-I_1 - 1.33I_2 = -.04 \therefore -4.33I_2 = -0.10$ $-I_3 - 2I_2 = -.06$		
Note: Credit is earned if students indicate they used a calculator function to solve the system of equations.		
For a correct answer with correct units		1 point
$-4.33I_2 = -0.10 \therefore I_2 = 0.023\text{ A}$		

Question 2 (continued)

(a) continued
iii. LO CNV-5.A.a, SP 6.C
1 point

Calculate the power dissipated by the $200\ \Omega$ resistor.

For using a correct equation to calculate the power in the $200\ \Omega$ resistor		1 point
$P = I^2R = (0.023\text{ A})^2(200\ \Omega)\ \therefore\ P = 0.107\text{ W}$		

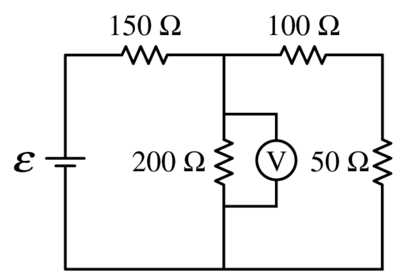


Figure 2

The two 6.0 V batteries are replaced with a battery with voltage $\mathcal{E}$ and a resistor of resistance $50\ \Omega$, as shown above. The voltmeter V shows that the voltage across the $200\ \Omega$ resistor is 4.4 V.

(b) LO CNV-6.C.a, SP 6.B, 6.C
2 points

Calculate the current through the $50\ \Omega$ resistor.

For correctly calculating the equivalent resistance of the branch with the $50\ \Omega$ resistor		1 point
$R = 100\ \Omega + 50\ \Omega = 150\ \Omega$		
For using the correct potential difference in Ohm's law to calculate the current in the $50\ \Omega$ resistor		1 point
$I = \frac{V}{R} = \frac{(4.4\text{ V})}{(100\ \Omega + 50\ \Omega)}\ \therefore\ I = 0.029\text{ A}$		

Question 2 (continued)

(c) LO CNV-6.C.a, SP 6.A, 6.C
3 points

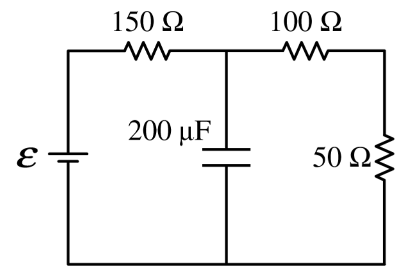
Calculate the voltage ε of the battery.

For using a correct equation to determine the current through the 150 Ω resistor		1 point
For correctly substituting the current from part (b) as I_3		1 point
$I_1 = I_2 + I_3 = \frac{(4.4\text{ V})}{(200\ \Omega)} + 0.029\text{ A} = 0.051\text{ A}$		
For using a correct equation to determine the emf of the battery		1 point
$\varepsilon = I_1 R_1 + V_2 = (0.051\text{ A})(150\ \Omega) + 4.4\text{ V} = 12.1\text{ V}$		
Alternate Third Point		
For calculating the equivalent resistance of the circuit and substituting this resistance into a correct equation to determine the emf of the battery		*1 point*
$R_T = 150\ \Omega + \frac{1}{\frac{1}{200\ \Omega} + \frac{1}{150\ \Omega}} = 236\ \Omega$		
$\varepsilon = I_1 R_T = (0.051\text{ A})(236\ \Omega) = 12.0\text{ V}$		
Alternate Solution		*Alternate Points*
For using a correct equation to determine the equivalent resistance of the parallel resistors		*1 point*
$\frac{1}{R_P} = \frac{1}{200\ \Omega} + \frac{1}{150\ \Omega}\ \therefore R_P = 86\ \Omega$		
For correctly substituting the given potential difference and the calculated equivalent resistance to determine the total current of the circuit		*1 point*
$I_T = \frac{(4.4\text{ V})}{(86\ \Omega)} = 0.051\text{ A}$		
For calculating the equivalent resistance of the circuit and substituting into a correct equation to determine the emf of the battery		*1 point*
$R_T = 150\ \Omega + 86\ \Omega = 236\ \Omega$		
$\varepsilon = I_1 R_T = (0.051\text{ A})(236\ \Omega) = 12.0\text{ V}$		

Question 2 (continued)

(d)

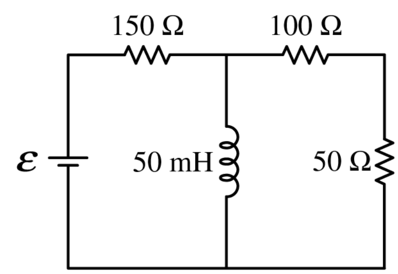
i. LO CNV-7.B.a, SP 6.A, 6.C
2 points



The 200 Ω resistor in the circuit in Figure 2 is replaced with a 200 μF capacitor, as shown on the right, and the circuit is allowed to reach steady state. Calculate the current through the 50 Ω resistor.

For substituting the voltage consistent with part (c) into Ohm's law		1 point
For correctly calculating the equivalent resistance of the circuit		1 point
$I = \frac{\mathcal{E}}{R_{tot}} = \frac{(12.1\text{ V})}{(150\ \Omega + 100\ \Omega + 50\ \Omega)} = 40.3\text{ mA}$		

ii. LO CNV-10.C.a, SP 7.A, 7.C
2 points



The 200 Ω resistor in the circuit in Figure 2 is replaced with an ideal 50 mH inductor, as shown on the right, and the circuit is allowed to reach steady state. Is the current in the 50 Ω resistor greater than, less than, or equal to the current calculated in part (b)?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For correctly selecting "Less than" with an attempt at a relevant justification		1 point
For a correct justification		1 point
Example: Because steady state is reached, the inductor will act as a short circuit. So all the current will pass through the inductor and no current will pass through the 50 Ω resistor.		

Question 2 (continued)

Learning Objectives

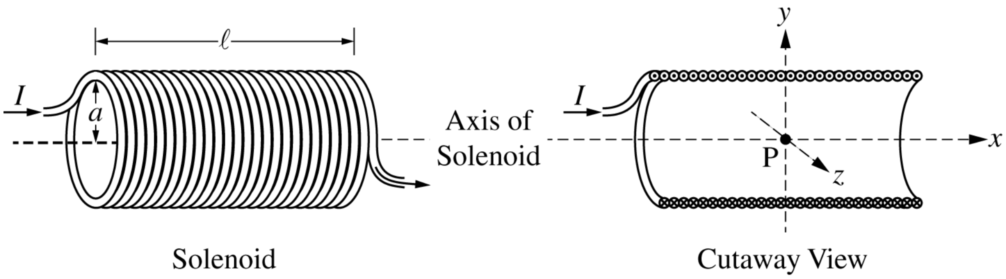
CNV-5.A.a: Derive expressions that relate current, voltage, and resistance to the rate at which heat is produced in a resistor.
CNV-6.C.a: Calculate voltage, current, and power dissipation for any resistor in a circuit containing a network of known resistors with a single battery or energy source.
CNV-6.F.b: Set up simultaneous equations to calculate at least two unknowns (currents or resistance values) in a multi-loop circuit.
CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-10.C.a: Calculate initial transient currents and final steady-state currents through any part of a series or parallel circuit containing an inductor and one or more resistors.

Science Practices

6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

Question 3

15 points



Note: Figures not drawn to scale.

A solenoid is used to generate a magnetic field. The solenoid has an inner radius a, length ℓ, and N total turns of wire. A power supply, not shown, is connected to the solenoid and generates current I, as shown in the figure on the left above. The x-axis runs along the axis of the solenoid. Point P is in the middle of the solenoid at the origin of the xyz-coordinate system, as shown in the cutaway view on the right above. Assume $\ell >> a$.

(a) LO CNV-8.E.a, SP 7.A, 7.C
2 points

Select the correct direction of the magnetic field at point P.

____ $+x$-direction ____ $+y$-direction ____ $+z$-direction
____ $-x$-direction ____ $-y$-direction ____ $-z$-direction

Justify your selection.

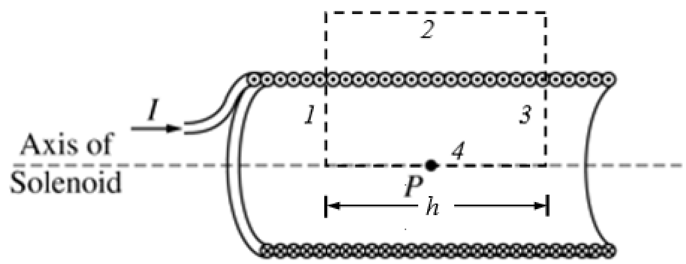
For choosing the "$+x$-direction" and providing a justification		1 point
For a correct justification		1 point
Example: Using the right-hand rule for current on the left side of the solenoid, the fingers curl into the loop, so the magnetic field points to the right, or in the $+x$-direction.		
Example: Using the right-hand rule for solenoids, when the fingers curl around the solenoid in the direction of the current, the thumb points to the right, therefore the magnetic field is to the right, or in the $+x$-direction.		

Question 3 (continued)

(b)

i. LO CNV-8.C.c, SP 3.D
1 point

On the cutaway view below, clearly draw an Amperian loop that can be used to determine the magnetic field at point P at the center of the solenoid.



For drawing a rectangle with one side along the central axis of the solenoid and another side outside the solenoid and whose edges do not extend beyond the solenoid		1 point

ii. LO CNV-8.C.c, SP 5.A, 5.E
2 points

Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of I, ℓ, N, a, and physical constants, as appropriate.

For using Ampere's law to calculate the magnetic field along the axis of the solenoid		1 point
$\int B \bullet d\ell = \mu_0 I_{enc} \therefore \left(\int B \bullet d\ell\right)_1 + \left(\int B \bullet d\ell\right)_2 + \left(\int B \bullet d\ell\right)_3 + \left(\int B \bullet d\ell\right)_4 = \mu_0 I_{enc}$		
$\left(\int B \bullet d\ell\right)_1 + 0 + \left(-\left(\int B \bullet d\ell\right)_1\right) + Bh = \mu_0 \frac{N}{\ell} hI$		
$Bh = \mu_0 \frac{N}{\ell} hI$		
For a correct answer		1 point
$B = \frac{\mu_0 NI}{\ell}$		

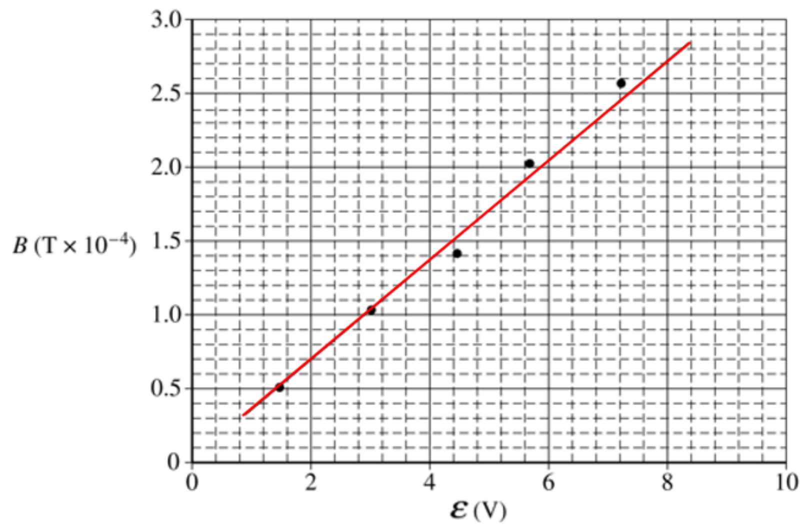
Some physics students conduct an experiment to determine the resistance R_S of a solenoid with radius $a = 0.015$ m, total turns $N = 100$, and total length $\ell = 0.40$ m. The students connect the solenoid to a variable power supply. A magnetic field sensor is used to measure the magnetic field strength along the central axis at the center of the solenoid. The plot of the magnetic field strength B as a function of the emf $\mathcal{E}$ of the power supply is shown below.

Question 3 (continued)

(c) LO CNV-8.C.c, SP 4.C

i. 1 point

On the graph above, draw a best-fit line for the data.



For drawing a best-fit line with at least one point above and one point below the line		1 point

ii. LO CNV-8.C.c, SP 6.B, 6.C

2 points

Use the straight line to determine the resistance R_S of the solenoid used in the experiment.

For calculating the slope using the best-fit line and not the data points unless they fall on the best-fit line		1 point
$slope = \dfrac{\Delta y}{\Delta x} = \dfrac{(2.5 - 0.9)(\times 10^{-4}\ \text{T})}{(6.4 - 2.0)\ \text{V}} = 0.36 \times 10^{-4} = 3.6 \times 10^{-5}\ \text{T/V}$		
For correctly giving the expression that relates the slope to the resistance of the solenoid		1 point
$B = \dfrac{\mu_0 NI}{\ell} = \dfrac{\mu_0 N\mathcal{E}}{\ell R}\ \therefore\ slope = \dfrac{\mu_0 N}{\ell R_S}\ \therefore\ R_S = \dfrac{\mu_0 N}{\ell \times slope}$ $R_S = \dfrac{(4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A})(100\ \text{turns})}{(0.40\ \text{m})(3.6 \times 10^{-5}\ \text{T/V})} = 8.7\ \Omega$		

One of the students notes that the horizontal component of the magnetic field of Earth is 2.5×10^{-5} T.

Question 3 (continued)

(d)

i. LO CNV-8.E.a, SP 2.D
1 point

Is there evidence from the graph that the horizontal orientation of the solenoid affects the measured values for *B*?

____ Yes ____ No

Justify your answer.

If the line on the graph does not go through the origin, select "Yes"		
For a correct justification		1 point
Example: The horizontal component of Earth's magnetic field will add or subtract from the magnetic field of the solenoid depending on the orientation of the solenoid.		
Alternate Solution		*Alternate Points*
If the line on the graph does pass through the origin, select "No"		
For a correct justification		*1 point*
Example: Based on the graph, the line passes through the origin, so the magnetic field is zero when the emf of the power supply is zero, therefore Earth's magnetic field is not affecting the values of B.		

ii. LO CNV-8.E.a, SP 2.E
1 point

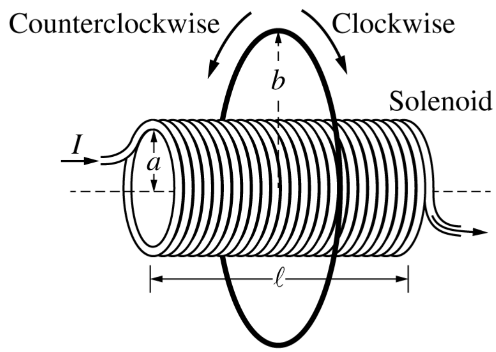
Would the horizontal orientation of the solenoid affect the calculated value for R_S ?

____ Yes ____ No

Justify your answer.

Select "No"		
For a correct justification		1 point
Example: The horizontal component of Earth's magnetic field will not affect the change in magnetic field as the emf is changed. Therefore, the value for the resistance of the solenoid will not change.		

Question 3 (continued)



A thin conducting loop of radius b and resistance R_L is placed concentric with the solenoid, as shown above. The current in the solenoid is decreased from I to zero over time Δt.

(e)

i. LO FIE-6.A.b, SP 7.A, 7.C
2 points

Is the direction of the induced current in the loop clockwise or counterclockwise during the time period that the current in the solenoid is decreasing?

____ Clockwise ____ Counterclockwise

Justify your answer.

Select "Clockwise"		
For a justification indicating that the magnetic field inside the solenoid, and therefore the loop, will decrease		1 point
For a justification using Lenz's law to relate the change in magnetic field to the direction of the current		1 point
Example: As the current in the solenoid decreases, the magnetic field inside the solenoid decreases. As the solenoid's magnetic field decreases, the induced current in the loop will create a magnetic field to oppose this change. Because the solenoid's magnetic field is toward the right and decreasing, the magnetic field due to the current in the loop must be toward the right. Therefore, the current in the loop must be clockwise.		

Question 3 (continued)

(e) continued

ii. LO FIE-6.A.d, SP 5.A, 5.E
3 points

Derive an equation for the average induced current i_{IND} in the loop during the time period that the current in the solenoid is decreasing. Express your answer in terms of I, ℓ, N, a, b, R_L, R_S, Δt, and physical constants, as appropriate.

For using Faraday's law to calculate the emf in the loop		1 point
$\varepsilon = \left\lvert \frac{d\Phi}{dt} \right\rvert = \frac{d(BA)}{dt} = A\frac{\Delta B}{\Delta t}$		
For using Ohm's law to calculate the current in the loop		1 point
$I_{IND} = \frac{V}{R} = \frac{A\Delta B/\Delta t}{R_L} = \frac{A\Delta B}{R_L \Delta t}$		
For using the correct radius for the area in the equation above		1 point
$I_{IND} = \frac{\pi a^2 \mu_0 N \Delta I/\ell}{R_L \Delta t} = \frac{\pi a^2 \mu_0 N I}{R_L \ell \Delta t}$		

Learning Objectives

CNV-8.C.c: Derive the expression for the magnetic field of an ideal solenoid (length dimension is much larger than the radius of the solenoid) using Ampère's law.
CNV-8.E.a: Describe the direction of a magnetic field at a point in space due to various combinations of conductors, wires, cylindrical conductors, or loops.
FIE-6.A.b: Describe the direction of an induced current in a conductive loop that is placed in a changing magnetic field.
FIE-6.A.d: Calculate the magnitude and direction of induced EMF and induced current in a conductive loop (or conductive bar) when the magnitude of either the field or area of loop is changing at a constant rate.

Science Practices

2.D: Make observations or collect data from representations of laboratory setups or results.
2.E: Identify or describe potential sources of experimental error.
3.D: Create appropriate diagrams to represent physical situations.
4.C: Linearize data and/or determine a best-fit line or curve.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.