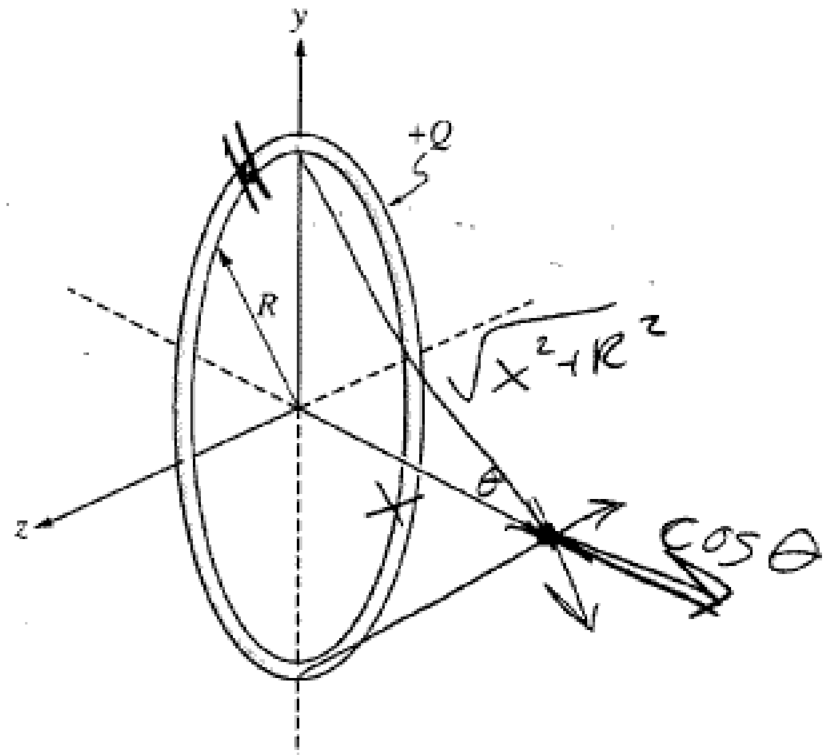
AP® Physics C: Electricity & Magnetism
1999 Sample Student Responses

These materials were produced by Educational Testing Service (ETS), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 3,900 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT™, the Advanced Placement Program® (AP®), and Pacesetter®. The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.



E&M 3. The nonconducting ring of radius R shown above lies in the yz-plane and carries a uniformly distributed positive charge Q.

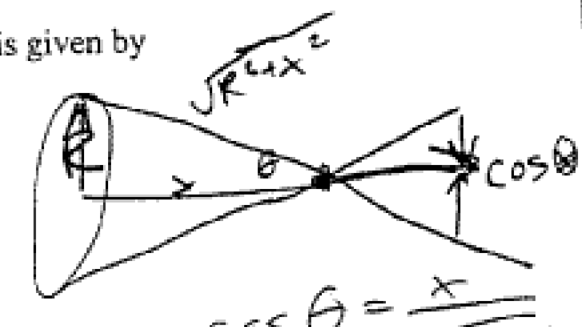
(a) Determine the electric potential at points along the x-axis as a function of x.

$$V = \int_0^Q \frac{k\,dq}{r}$$

$$= \frac{kQ}{\sqrt{x^2+R^2}}$$

(b) i. Show that the x-component of the electric field along the x-axis is given by

$$E_x = \frac{Qx}{4\pi\epsilon_0 (R^2 + x^2)^{\frac{3}{2}}}.$$



$$E = \int_0^Q \frac{k\,dq}{r^2} \cos\theta$$

$$= \frac{k}{R^2} \cos\theta \int_0^Q dq$$

$$= \frac{k}{R^2} \left(\frac{x}{\sqrt{R^2+x^2}} \right) Q$$

$$= \frac{k}{(\sqrt{R^2+x^2})^2} \cdot \frac{x}{\sqrt{R^2+x^2}} \cdot Q = \frac{Qx}{4\pi\varepsilon_0 (R^2+x^2)^{3/2}}$$

$$\cos\theta = \frac{x}{\sqrt{R^2+x^2}}$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

ii. What are the y- and z- components of the electric field along the x-axis?

there are no z-components, and the y-components are equal and opposite, so they cancel out

(c) Determine the following.

i. The value of x for which E_x is a maximum

deriv. of Ex $\rightarrow E_x' = 0$

$u = R^2+x^2$ $du = 2x\,dx$

$$E_x = \frac{kQx}{(R^2+x^2)^{3/2}}$$

$$E'_x = \frac{(R^2+x^2)^{3/2} kQ - kQx \frac{3}{2}(R^2+x^2)^{1/2}(2x)}{(R^2+x^2)^3}$$

$$0 = \frac{\left((R^2+x^2)^{3/2} - 3x^2(R^2+x^2)^{1/2}\right) \cancel{kQ}}{\cancel{(R^2+x^2)^3}}$$

$$0 = (R^2+x^2)^{1/2} \left((R^2+x^2)^1 - 3x^2 \right)$$

$$3x^2 = (R^2+x^2)^1$$

$$3x^2 = R^2 + x^2$$

$$2x^2 = R^2$$

$$x^2 = \frac{R^2}{2}$$

$$x = \frac{R}{\sqrt{2}}$$

ii. The maximum electric field $E_{x\,\max}$

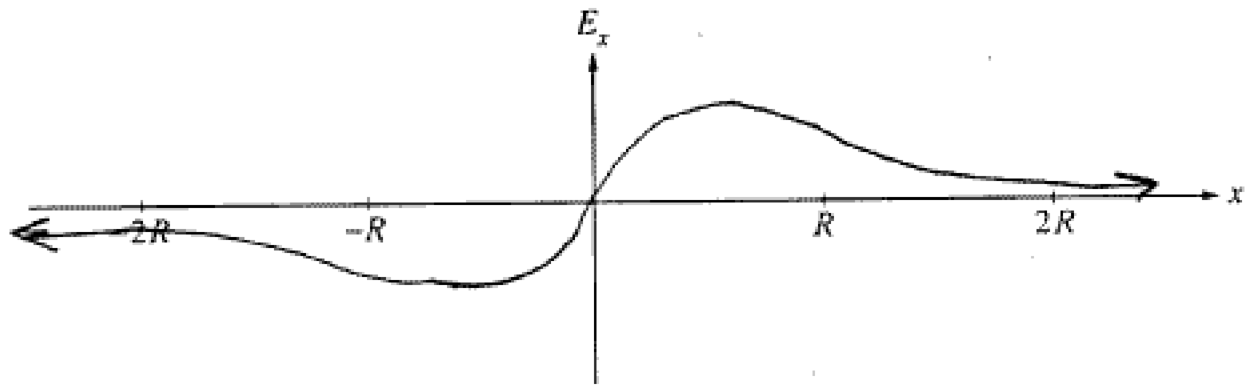
$$E_x\left(\frac{R}{\sqrt{2}}\right) = \frac{Q\left(\frac{R}{\sqrt{2}}\right)}{4\pi\varepsilon_0 \left(R^2 + \left(\frac{R}{\sqrt{2}}\right)^2\right)^{3/2}}$$

$$= \frac{QR}{\sqrt{2}\, 4\pi\varepsilon_0 (R^2 + \frac{1}{2}R^2)^{3/2}}$$

$$= \frac{QR}{4\pi\varepsilon_0 (\sqrt{2}) (\frac{3}{2}R^2)^{3/2}}$$

GO ON TO THE NEXT PAGE

(d) On the axes below, sketch E_x *versus* x for points on the x-axis from $x = -2R$ to $x = +2R$.



(e) An electron is placed at $x = R/2$ and released from rest. Qualitatively describe its subsequent motion.

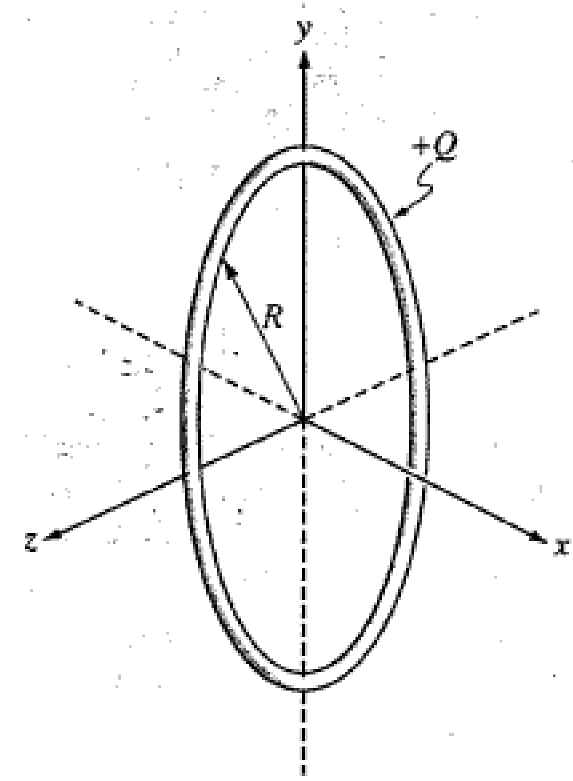
It will oscillate btwn $x = -R/2$ & $x = R/2$ on the x-axis,

STOP

END OF SECTION II, ELECTRICITY AND MAGNETISM

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON SECTION II, ELECTRICITY AND MAGNETISM, ONLY. DO NOT TURN TO ANY OTHER TEST MATERIALS.

- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK COVER OF THIS BOOKLET.
- CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE TWO BOXES ON THE BACK COVER (TOP LEFT AND RIGHT).
- MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.



E&M 3. The nonconducting ring of radius R shown above lies in the yz-plane and carries a uniformly distributed positive charge Q.

(a) Determine the electric potential at points along the x-axis as a function of x.

$$V=-\int_{\infty}^{x} E\cdot ds \qquad E=\int_0^{2\pi}\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,d\theta$$

$$E=\frac{Qx}{4\pi\varepsilon_0\,(x^2+R^2)^{\frac{3}{2}}}$$

$$V=-\int_{\infty}^{x}\frac{Qx}{4\pi\varepsilon_0\,(x^2+R^2)^{\frac{3}{2}}}\,dx$$

$$V=-\frac{Q}{4\pi\varepsilon_0}\frac{1}{2}\int_{\infty}^{x} u^{-\frac{3}{2}}\,du \qquad u=x^2+R^2 \qquad du=2x\,dx$$

$$V=-\frac{Q}{8\pi\varepsilon_0}\left[-2u^{-\frac{1}{2}}\right]_{\infty}^{x}$$

$$V=\frac{Q}{4\pi\varepsilon_0}\left[(x^2+R^2)^{-\frac{1}{2}}\right]_{\infty}^{x}$$

$$V=\frac{Q}{4\pi\varepsilon_0\,(x^2+R^2)^{\frac{1}{2}}}$$

$$V=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

$$V=\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+R^2}}$$

(b) i. Show that the x-component of the electric field along the x-axis is given by

$$E_x = \frac{Qx}{4\pi\epsilon_0(R^2 + x^2)^{\frac{3}{2}}}.$$

$$E_x = -\frac{dV}{dx}$$

$$E_x = -\frac{Q}{4\pi\epsilon_0}\frac{d}{dx}\left(\frac{1}{\sqrt{x^2+R^2}}\right)$$

$$E_x = \frac{Q}{4\pi\epsilon_0}\cdot -\frac{1}{2}(x^2+R^2)^{-\frac{3}{2}}\cdot 2x$$

$$E_x = \frac{Qx}{4\pi\epsilon_0(x^2+R^2)^{\frac{3}{2}}}$$

ii. What are the y- and z- components of the electric field along the x-axis?

zero

(c) Determine the following.

i. The value of x for which E_x is a maximum

at $x = \frac{\sqrt{2}}{2}R$

ii. The maximum electric field $E_{x\,max}$.

$$E_{xmax} = \frac{Q\cdot\frac{\sqrt{2}}{2}R}{4\pi\epsilon_0\left(\frac{1}{2}R^2+R^2\right)^{\frac{3}{2}}}$$

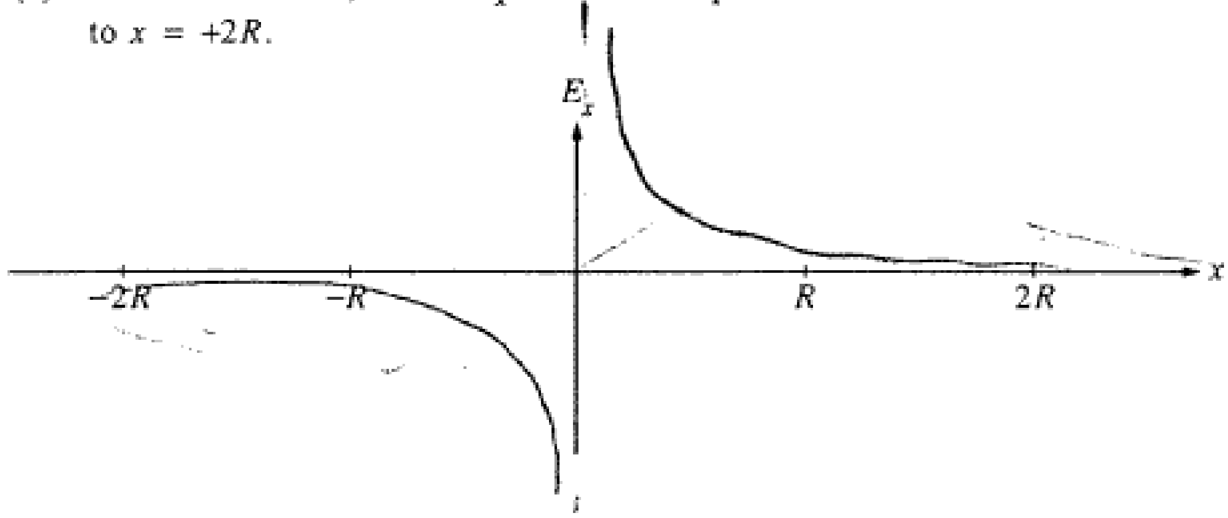
$$E_{xmax} = \frac{Q\cdot R}{4\pi\epsilon_0\left(\frac{3}{2}R^2\right)^{\frac{3}{2}}}$$

$$E_{xmax} = \frac{Q\cdot R}{4\pi\epsilon_0 R^3\left(\frac{3}{2}\right)^{\frac{3}{2}}}$$

$$E_{xmax} = \frac{Q}{4\pi\epsilon_0 R^2\left(\frac{3}{2}\right)^{\frac{3}{2}}}$$

GO ON TO THE NEXT PAGE

(d) On the axes below, sketch E_x *versus* x for points on the x-axis from $x = -2R$ to $x = +2R$.



(e) An electron is placed at $x = R/2$ and released from rest. Qualitatively describe its subsequent motion.

~~it initially accellerates toward the charged ring, passes through,~~

Simple harmonic motion with amplitude $\frac{R}{2}$ and equilibrium point at the center of the ring

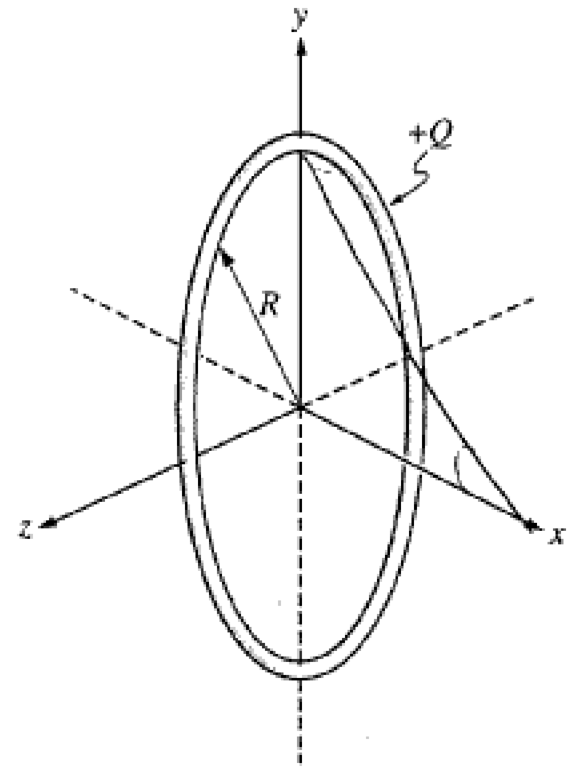
– so it initially accelerates toward the charged ring

STOP

END OF SECTION II, ELECTRICITY AND MAGNETISM

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON SECTION II, ELECTRICITY AND MAGNETISM, ONLY. DO NOT TURN TO ANY OTHER TEST MATERIALS.

- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK COVER OF THIS BOOKLET.
- CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE TWO BOXES ON THE BACK COVER (TOP LEFT AND RIGHT).
- MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.



E&M 3. The nonconducting ring of radius R shown above lies in the yz-plane and carries a uniformly distributed positive charge Q.

(a) Determine the electric potential at points along the x-axis as a function of x.

$$V = \frac{1}{4\pi\epsilon_0}\int_0^x \frac{q_0}{r^2}\,dr$$

$$V = \frac{q_0}{4\pi\epsilon_0\sqrt{x^2+R^2}}$$

(b) i. Show that the x-component of the electric field along the x-axis is given by

$$E_x = \frac{Qx}{4\pi\epsilon_0(R^2 + x^2)^{\frac{3}{2}}}.$$

$$E_x = \frac{-dV}{dr}$$

ii. What are the y- and z- components of the electric field along the x-axis?

$$\frac{E_y}{E_x} = \frac{R}{x}$$

$$\frac{E_z}{E_x} = \frac{R}{y}$$

$$E_y = (E_x)\frac{R}{x}$$

$$E_z = (E_x)\frac{R}{x}$$

(c) Determine the following.

i. The value of x for which E_x is a maximum

$$0 = \frac{dE_x}{dx} = -\frac{Q(2x^2 - r^2)}{}$$

$$0 = -Q(2x^2 - R^2)$$

$$0 = 2x^2 - R^2$$

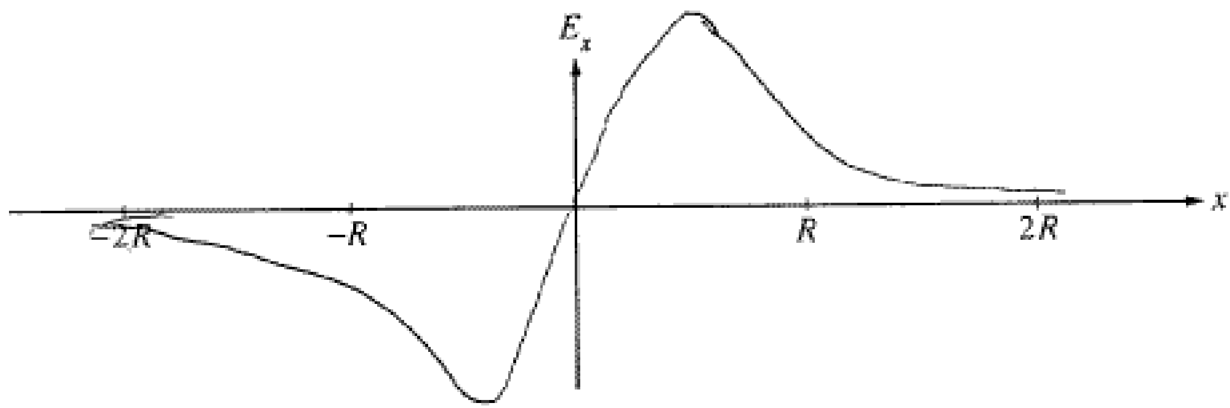
$$R^2 = 2x^2 \quad \sqrt{\frac{R^2}{2}} = x = \frac{R}{\sqrt{2}}$$

ii. The maximum electric field $E_{x\,max}$

$$E_{x\,max} = \frac{Q\frac{R}{\sqrt{2}}}{4\pi\epsilon_0\left(R^2 + \frac{R^2}{2}\right)^{\frac{3}{2}}}$$

GO ON TO THE NEXT PAGE

(d) On the axes below, sketch E_x *versus* x for points on the x-axis from $x = -2R$ to $x = +2R$.



(e) An electron is placed at $x = R/2$ and released from rest. Qualitatively describe its subsequent motion.

It will away from center of ring to the right

STOP

END OF SECTION II, ELECTRICITY AND MAGNETISM

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON SECTION II, ELECTRICITY AND MAGNETISM, ONLY. DO NOT TURN TO ANY OTHER TEST MATERIALS.

- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK COVER OF THIS BOOKLET.
- CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE TWO BOXES ON THE BACK COVER (TOP LEFT AND RIGHT).
- MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON <u>ALL</u> AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.