



AP[®] Calculus AB 1998 Scoring Guidelines

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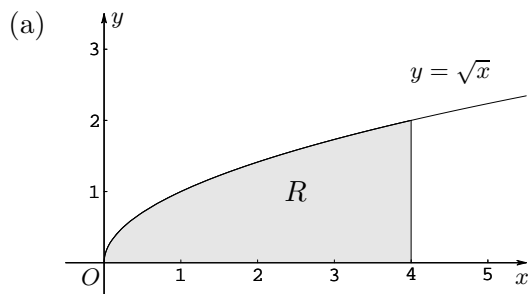
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1. Let R be the region bounded by the x -axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.
- Find the area of the region R .
 - Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.
 - Find the volume of the solid generated when R is revolved about the x -axis.
 - The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x -axis, they generate solids with equal volumes. Find the value of k .



$$A = \int_0^4 \sqrt{x} \, dx = \frac{2}{3} x^{3/2} \Big|_0^4 = \frac{16}{3} \text{ or } 5.333$$

(b)

$$\int_0^h \sqrt{x} \, dx = \frac{8}{3} \quad \text{---or---} \quad \int_0^h \sqrt{x} \, dx = \int_h^4 \sqrt{x} \, dx$$

$$\frac{2}{3} h^{3/2} = \frac{8}{3} \quad \text{---or---} \quad \frac{2}{3} h^{3/2} = \frac{16}{3} - \frac{2}{3} h^{3/2}$$

$$h = \sqrt[3]{16} \text{ or } 2.520 \text{ or } 2.519$$

(c)

$$V = \pi \int_0^4 (\sqrt{x})^2 \, dx = \pi \frac{x^2}{2} \Big|_0^4 = 8\pi$$

$$\text{or } 25.133 \text{ or } 25.132$$

(d)

$$\pi \int_0^k (\sqrt{x})^2 \, dx = 4\pi \quad \text{---or---} \quad \pi \int_0^k (\sqrt{x})^2 \, dx = \pi \int_k^4 (\sqrt{x})^2 \, dx$$

$$\pi \frac{k^2}{2} = 4\pi \quad \text{---or---} \quad \pi \frac{k^2}{2} = 8\pi - \pi \frac{k^2}{2}$$

$$k = \sqrt{8} \text{ or } 2.828$$

$$2 \left\{ \begin{array}{l} 1: A = \int_0^4 \sqrt{x} \, dx \\ 1: \text{answer} \end{array} \right.$$

$$2 \left\{ \begin{array}{l} 1: \text{equation in } h \\ 1: \text{answer} \end{array} \right.$$

$$3 \left\{ \begin{array}{l} 1: \text{limits and constant} \\ 1: \text{integrand} \\ 1: \text{answer} \end{array} \right.$$

$$2 \left\{ \begin{array}{l} 1: \text{equation in } k \\ 1: \text{answer} \end{array} \right.$$

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2. Let f be the function given by $f(x) = 2xe^{2x}$.
- Find $\lim_{x \rightarrow -\infty} f(x)$ and $\lim_{x \rightarrow \infty} f(x)$.
 - Find the absolute minimum value of f . Justify that your answer is an absolute minimum.
 - What is the range of f ?
 - Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b .
-

(a) $\lim_{x \rightarrow -\infty} 2xe^{2x} = 0$

$\lim_{x \rightarrow \infty} 2xe^{2x} = \infty$ or DNE

$$2 \left\{ \begin{array}{l} 1: 0 \text{ as } x \rightarrow -\infty \\ 1: \infty \text{ or DNE as } x \rightarrow \infty \end{array} \right.$$

(b) $f'(x) = 2e^{2x} + 2x \cdot 2 \cdot e^{2x} = 2e^{2x}(1 + 2x) = 0$

if $x = -1/2$

$f(-1/2) = -1/e$ or -0.368 or -0.367

$-1/e$ is an absolute minimum value because:

(i) $f'(x) < 0$ for all $x < -1/2$ and
 $f'(x) > 0$ for all $x > -1/2$

–or–

(ii) $f'(x) \begin{array}{c} - \qquad \qquad \qquad + \\ \hline \qquad \qquad \qquad | \qquad \qquad \qquad \\ \qquad \qquad \qquad -1/2 \end{array}$

and $x = -1/2$ is the only critical number

$$3 \left\{ \begin{array}{l} 1: \text{ solves } f'(x) = 0 \\ 1: \text{ evaluates } f \text{ at student's critical point} \\ \quad 0/1 \text{ if not local minimum from} \\ \quad \text{student's derivative} \\ 1: \text{ justifies absolute minimum value} \\ \quad 0/1 \text{ for a local argument} \\ \quad 0/1 \text{ without explicit symbolic} \\ \quad \text{derivative} \end{array} \right.$$

Note: 0/3 if no absolute minimum based on student's derivative

(c) Range of $f = [-1/e, \infty)$
or $[-0.367, \infty)$
or $[-0.368, \infty)$

1: answer

Note: must include the left-hand endpoint; exclude the right-hand “endpoint”

(d) $y' = be^{bx} + b^2xe^{bx} = be^{bx}(1 + bx) = 0$

if $x = -1/b$

At $x = -1/b, y = -1/e$

y has an absolute minimum value of $-1/e$ for all nonzero b

$$3 \left\{ \begin{array}{l} 1: \text{ sets } y' = be^{bx}(1 + bx) = 0 \\ 1: \text{ solves student's } y' = 0 \\ 1: \text{ evaluates } y \text{ at a critical number} \\ \quad \text{and gets a value independent of } b \end{array} \right.$$

Note: 0/3 if only considering specific values of b

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AB 2 Board Note # 1

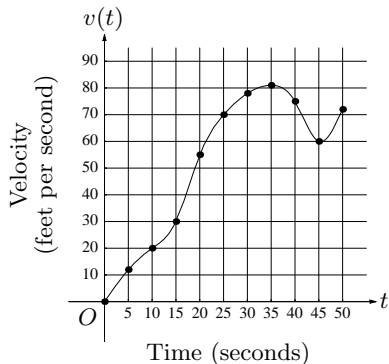
Part (d)

3/3 Argument with the following three ingredients:

1. The graph of $y = bxe^{bx}$ is a horizontal compression or expansion (with a reflection across the y -axis if $b < 0$) of the graph of $y = xe^x$.
2. The range of $y = bxe^{bx}$ is therefore the same as the range of $y = xe^x$.
3. Therefore the absolute minimum value of $y = bxe^{bx}$ is the same for all (non-zero) values of b .

0/3 Analyzing the horizontal compression/expansion of graphs of $y = bxe^{bx}$ for specific values of b .

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t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t , is shown to the right of the graph.
- During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
 - Find the average acceleration of the car, in ft/sec^2 , over the interval $0 \leq t \leq 50$.
 - Find one approximation for the acceleration of the car, in ft/sec^2 , at $t = 40$. Show the computations you used to arrive at your answer.
 - Approximate $\int_0^{50} v(t) dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

- (a) Acceleration is positive on $(0, 35)$ and $(45, 50)$ because the velocity $v(t)$ is increasing on $[0, 35]$ and $[45, 50]$

3 { 1: (0, 35)
1: (45, 50)
1: reason

Note: ignore inclusion of endpoints

- (b) Avg. Acc. = $\frac{v(50) - v(0)}{50 - 0} = \frac{72 - 0}{50} = \frac{72}{50}$
or 1.44 ft/sec^2

1: answer

- (c) Difference quotient; e.g.

$$\frac{v(45) - v(40)}{5} = \frac{60 - 75}{5} = -3 \text{ ft/sec}^2 \text{ or}$$

$$\frac{v(40) - v(35)}{5} = \frac{75 - 81}{5} = -\frac{6}{5} \text{ ft/sec}^2 \text{ or}$$

$$\frac{v(45) - v(35)}{10} = \frac{60 - 81}{10} = -\frac{21}{10} \text{ ft/sec}^2$$

–or–

Slope of tangent line, e.g.

through $(35, 90)$ and $(40, 75)$: $\frac{90 - 75}{35 - 40} = -3 \text{ ft/sec}^2$

2 { 1: method
1: answer

Note: 0/2 if first point not earned

- (d) $\int_0^{50} v(t) dt$
 $\approx 10[v(5) + v(15) + v(25) + v(35) + v(45)]$
 $= 10(12 + 30 + 70 + 81 + 60)$
 $= 2530 \text{ feet}$

3 { 1: midpoint Riemann sum
1: answer
1: meaning of integral

This integral is the total distance traveled in feet over the time 0 to 50 seconds.

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4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2 + 1}{2y}$.
- (a) Find the slope of the graph of f at the point where $x = 1$.
- (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.
- (c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$ with the initial condition $f(1) = 4$.
- (d) Use your solution from part (c) to find $f(1.2)$.

(a) $\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$

$$\left. \frac{dy}{dx} \right|_{\substack{x=1 \\ y=4}} = \frac{3+1}{2 \cdot 4} = \frac{4}{8} = \frac{1}{2}$$

(b) $y - 4 = \frac{1}{2}(x - 1)$

$$f(1.2) - 4 \approx \frac{1}{2}(1.2 - 1)$$

$$f(1.2) \approx 0.1 + 4 = 4.1$$

(c) $2y \, dy = (3x^2 + 1) \, dx$

$$\int 2y \, dy = \int (3x^2 + 1) \, dx$$

$$y^2 = x^3 + x + C$$

$$4^2 = 1 + 1 + C$$

$$14 = C$$

$$y^2 = x^3 + x + 14$$

$$y = \sqrt{x^3 + x + 14} \text{ is branch with point } (1, 4)$$

$$f(x) = \sqrt{x^3 + x + 14}$$

(d) $f(1.2) = \sqrt{1.2^3 + 1.2 + 14} \approx 4.114$

1: answer

2 { 1: equation of tangent line
1: uses equation to approximate $f(1.2)$

5 { 1: separates variables
1: antiderivative of dy term
1: antiderivative of dx term
1: uses $y = 4$ when $x = 1$ to pick one function out of a family of functions
1: solves for y
0/1 if solving a linear equation in y
0/1 if no constant of integration

Note: max 0/5 if no separation of variables

Note: max 1/5 [1-0-0-0-0] if substitutes value(s) for x , y , or dy/dx before antidifferentiation

1: answer, from student's solution to the given differential equation in (c)

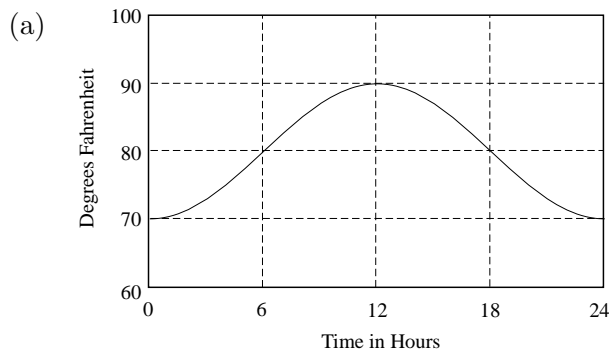
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5. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10 \cos\left(\frac{\pi t}{12}\right), \quad 0 \leq t \leq 24,$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

- Sketch the graph of F on the grid below.
- Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.
- An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?
- The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?



(b)
$$\begin{aligned} \text{Avg.} &= \frac{1}{14 - 6} \int_6^{14} \left[80 - 10 \cos\left(\frac{\pi t}{12}\right) \right] dt \\ &= \frac{1}{8} (697.2957795) \\ &= 87.162 \text{ or } 87.161 \\ &\approx 87^\circ \text{ F} \end{aligned}$$

(c)
$$\begin{aligned} \left[80 - 10 \cos\left(\frac{\pi t}{12}\right) \right] - 78 &\geq 0 \\ 2 - 10 \cos\left(\frac{\pi t}{12}\right) &\geq 0 \\ \left. \begin{array}{l} 5.230 \\ \text{or} \\ 5.231 \end{array} \right\} \leq t \leq \left\{ \begin{array}{l} 18.769 \\ \text{or} \\ 18.770 \end{array} \right. \end{aligned}$$

(d)
$$\begin{aligned} C &= 0.05 \int_{\substack{5.231 \\ \text{or} \\ 5.230}}^{\substack{18.770 \\ \text{or} \\ 18.769}} \left(\left[80 - 10 \cos\left(\frac{\pi t}{12}\right) \right] - 78 \right) dt \\ &= 0.05(101.92741) = 5.096 \approx \$5.10 \end{aligned}$$

1: bell-shaped graph
 minimum 70 at $t = 0, t = 24$ only
 maximum 90 at $t = 12$ only

3 { 2: integral
 1: limits and $1/(14 - 6)$
 1: integrand
 1: answer
 0/1 if integral not of the form
 $\frac{1}{b - a} \int_a^b F(t) dt$

2 { 1: inequality or equation
 1: solutions with interval

3 { 2: integral
 1: limits and 0.05
 1: integrand
 1: answer
 0/1 if integral not of the form
 $k \int_a^b (F(t) - 78) dt$

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6. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.

(b) Write an equation of each horizontal tangent line to the curve.

(c) The line through the origin with slope -1 is tangent to the curve at point P . Find the x - and y -coordinates of point P .

(a) $6y^2 \frac{dy}{dx} + 6x^2 \frac{dy}{dx} + 12xy - 24x + 6 \frac{dy}{dx} = 0$

$$\frac{dy}{dx}(6y^2 + 6x^2 + 6) = 24x - 12xy$$

$$\frac{dy}{dx} = \frac{24x - 12xy}{6x^2 + 6y^2 + 6} = \frac{4x - 2xy}{x^2 + y^2 + 1}$$

(b) $\frac{dy}{dx} = 0$

$$4x - 2xy = 2x(2 - y) = 0$$

$$x = 0 \text{ or } y = 2$$

$$\text{When } x = 0, \quad 2y^3 + 6y = 1; \quad y = 0.165$$

There is no point on the curve with y coordinate of 2.

$y = 0.165$ is the equation of the only horizontal tangent line.

(c) $y = -x$ is equation of the line.

$$2(-x)^3 + 6x^2(-x) - 12x^2 + 6(-x) = 1$$

$$-8x^3 - 12x^2 - 6x - 1 = 0$$

$$x = -1/2, \quad y = 1/2$$

-or-

$$\frac{dy}{dx} = -1$$

$$4x - 2xy = -x^2 - y^2 - 1$$

$$4x + 2x^2 = -x^2 - x^2 - 1$$

$$4x^2 + 4x + 1 = 0$$

$$x = -1/2, \quad y = 1/2$$

2 { 1: implicit differentiation
1: verifies expression for $\frac{dy}{dx}$

4 { 1: sets $\frac{dy}{dx} = 0$
1: solves $\frac{dy}{dx} = 0$
1: uses solutions for x to find equations of horizontal tangent lines
1: verifies which solutions for y yield equations of horizontal tangent lines

Note: max 1/4 [1-0-0-0] if $dy/dx = 0$ is not of the form $g(x, y)/h(x, y) = 0$ with solutions for both x and y

3 { 1: $y = -x$
1: substitutes $y = -x$ into equation of curve
1: solves for x and y

-or-

3 { 1: sets $\frac{dy}{dx} = -1$
1: substitutes $y = -x$ into $\frac{dy}{dx}$
1: solves for x and y

Note: max 2/3 [1-1-0] if importing incorrect derivative from part (a)