AP® Calculus AB
1998 Scoring Guidelines

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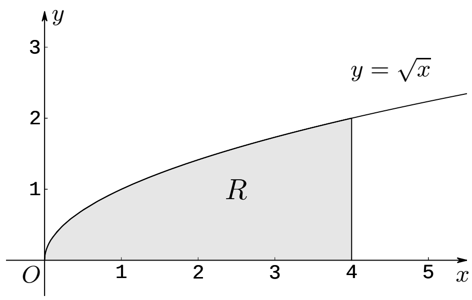
1. Let R be the region bounded by the x–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.
   (a) Find the area of the region R.
   (b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.
   (c) Find the volume of the solid generated when R is revolved about the x–axis.
   (d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

(a)



$$A = \int_0^4 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{16}{3} \quad \text{or} \quad 5.333$$

2 $\begin{cases} 1: & A = \int_0^4 \sqrt{x}\,dx \\ 1: & \text{answer} \end{cases}$

(b) $\displaystyle\int_0^h \sqrt{x}\,dx = \frac{8}{3}$ –or– $\displaystyle\int_0^h \sqrt{x}\,dx = \int_h^4 \sqrt{x}\,dx$

$\displaystyle\frac{2}{3}h^{3/2} = \frac{8}{3}$ | $\displaystyle\frac{2}{3}h^{3/2} = \frac{16}{3} - \frac{2}{3}h^{3/2}$

$h = \sqrt[3]{16}$ or 2.520 or 2.519

2 $\begin{cases} 1: & \text{equation in } h \\ 1: & \text{answer} \end{cases}$

(c) $$V = \pi \int_0^4 (\sqrt{x})^2\,dx = \pi \frac{x^2}{2}\Big|_0^4 = 8\pi$$

or 25.133 or 25.132

3 $\begin{cases} 1: & \text{limits and constant} \\ 1: & \text{integrand} \\ 1: & \text{answer} \end{cases}$

(d) $\displaystyle\pi \int_0^k (\sqrt{x})^2\,dx = 4\pi$ –or– $\displaystyle\pi \int_0^k (\sqrt{x})^2\,dx = \pi \int_k^4 (\sqrt{x})^2\,dx$

$\displaystyle\pi \frac{k^2}{2} = 4\pi$ | $\displaystyle\pi \frac{k^2}{2} = 8\pi - \pi \frac{k^2}{2}$

$k = \sqrt{8}$ or 2.828

2 $\begin{cases} 1: & \text{equation in } k \\ 1: & \text{answer} \end{cases}$

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2. Let f be the function given by $f(x) = 2xe^{2x}$.

   (a) Find $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to\infty} f(x)$.

   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

   (c) What is the range of f?

   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

(a) $\lim\limits_{x\to-\infty} 2xe^{2x} = 0$

$\lim\limits_{x\to\infty} 2xe^{2x} = \infty$ or DNE

$$2\begin{cases} 1: & 0 \text{ as } x \to -\infty \\ 1: & \infty \text{ or DNE as } x \to \infty \end{cases}$$

(b) $f'(x) = 2e^{2x} + 2x \cdot 2 \cdot e^{2x} = 2e^{2x}(1 + 2x) = 0$

if $x = -1/2$

$f(-1/2) = -1/e$ or -0.368 or -0.367

$-1/e$ is an absolute minimum value because:

(i) $f'(x) < 0$ for all $x < -1/2$ and
$f'(x) > 0$ for all $x > -1/2$

–or–

(ii)
$$\begin{array}{c|cc} f'(x) & - & + \\ \hline & \multicolumn{2}{c}{-1/2} \end{array}$$

and $x = -1/2$ is the only critical number

3
- 1: solves $f'(x) = 0$
- 1: evaluates f at student's critical point
  0/1 if not local minimum from student's derivative
- 1: justifies absolute minimum value
  0/1 for a local argument
  0/1 without explicit symbolic derivative

Note: 0/3 if no absolute minimum based on student's derivative

(c) Range of $f = [-1/e, \infty)$
or $[-0.367, \infty)$
or $[-0.368, \infty)$

1: answer

Note: must include the left–hand endpoint; exclude the right–hand "endpoint"

(d) $y' = be^{bx} + b^2xe^{bx} = be^{bx}(1 + bx) = 0$

if $x = -1/b$

At $x = -1/b$, $y = -1/e$

y has an absolute minimum value of $-1/e$ for all nonzero b

3
- 1: sets $y' = be^{bx}(1 + bx) = 0$
- 1: solves student's $y' = 0$
- 1: evaluates y at a critical number and gets a value independent of b

Note: 0/3 if only considering specific values of b

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AB 2 Board Note # 1

Part (d)

3/3 Argument with the following three ingredients:

1. The graph of $y = bxe^{bx}$ is a horizontal compression or expansion (with a reflection across the y–axis if $b < 0$) of the graph of $y = xe^x$.
2. The range of $y = bxe^{bx}$ is therefore the same as the range of $y = xe^x$.
3. Therefore the absolute minimum value of $y = bxe^{bx}$ is the same for all (non–zero) values of b.

0/3 Analyzing the horizontal compression/expansion of graphs of $y = bxe^{bx}$ for specific values of b.

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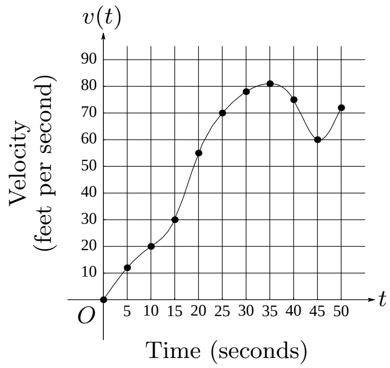

t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

(a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

(b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

(c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

(d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

(a) Acceleration is positive on $(0, 35)$ and $(45, 50)$ because the velocity $v(t)$ is increasing on $[0, 35]$ and $[45, 50]$

$$\mathbf{3}\begin{cases} 1: & (0,35) \\ 1: & (45,50) \\ 1: & \text{reason} \end{cases}$$

Note: ignore inclusion of endpoints

(b) Avg. Acc. $= \dfrac{v(50) - v(0)}{50 - 0} = \dfrac{72 - 0}{50} = \dfrac{72}{50}$

or $\quad 1.44$ ft/sec^2

1: answer

(c) Difference quotient; e.g.

$$\frac{v(45) - v(40)}{5} = \frac{60 - 75}{5} = -3 \text{ ft/sec}^2 \quad \text{or}$$

$$\frac{v(40) - v(35)}{5} = \frac{75 - 81}{5} = -\frac{6}{5} \text{ ft/sec}^2 \quad \text{or}$$

$$\frac{v(45) - v(35)}{10} = \frac{60 - 81}{10} = -\frac{21}{10} \text{ ft/sec}^2$$

–or–

Slope of tangent line, e.g.

through $(35, 90)$ and $(40, 75)$: $\dfrac{90 - 75}{35 - 40} = -3$ ft/sec^2

$$\mathbf{2}\begin{cases} 1: & \text{method} \\ 1: & \text{answer} \end{cases}$$

Note: 0/2 if first point not earned

(d) $\displaystyle\int_0^{50} v(t)\,dt$

$\approx 10[v(5) + v(15) + v(25) + v(35) + v(45)]$

$= 10(12 + 30 + 70 + 81 + 60)$

$= 2530$ feet

This integral is the total distance traveled in feet over the time 0 to 50 seconds.

$$\mathbf{3}\begin{cases} 1: & \text{midpoint Riemann sum} \\ 1: & \text{answer} \\ 1: & \text{meaning of integral} \end{cases}$$

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4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$.

   (a) Find the slope of the graph of f at the point where $x = 1$.

   (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

   (c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.

   (d) Use your solution from part (c) to find $f(1.2)$.

(a) $\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$

$$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=4}} = \frac{3+1}{2\cdot 4} = \frac{4}{8} = \frac{1}{2}$$

1: answer

(b) $y - 4 = \dfrac{1}{2}(x-1)$

$f(1.2) - 4 \approx \dfrac{1}{2}(1.2-1)$

$f(1.2) \approx 0.1 + 4 = 4.1$

2 $\begin{cases} 1: & \text{equation of tangent line} \\ 1: & \text{uses equation to approximate } f(1.2) \end{cases}$

(c) $2y\,dy = (3x^2+1)\,dx$

$\int 2y\,dy = \int (3x^2+1)\,dx$

$y^2 = x^3 + x + C$

$4^2 = 1 + 1 + C$

$14 = C$

$y^2 = x^3 + x + 14$

$y = \sqrt{x^3+x+14}$ is branch with point $(1, 4)$

$f(x) = \sqrt{x^3+x+14}$

5 $\begin{cases} 1: & \text{separates variables} \\ 1: & \text{antiderivative of } dy \text{ term} \\ 1: & \text{antiderivative of } dx \text{ term} \\ 1: & \text{uses } y = 4 \text{ when } x = 1 \text{ to pick one function out of a family of functions} \\ 1: & \text{solves for } y \\ & 0/1 \text{ if solving a linear equation in } y \\ & 0/1 \text{ if no constant of integration} \end{cases}$

Note: max 0/5 if no separation of variables

Note: max 1/5 [1-0-0-0-0] if substitutes value(s) for x, y, or dy/dx before antidifferentiation

(d) $f(1.2) = \sqrt{1.2^3 + 1.2 + 14} \approx 4.114$

1: answer, from student's solution to the given differential equation in (c)

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5. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right),\ 0 \le t \le 24,$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

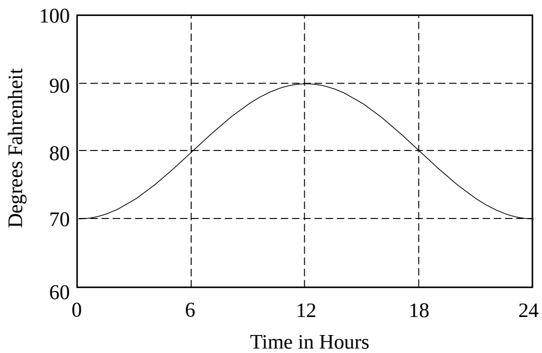
(a) Sketch the graph of F on the grid below.

(b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

(c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

(d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24–hour period?

(a)



1: bell–shaped graph
minimum 70 at $t = 0$, $t = 24$ only
maximum 90 at $t = 12$ only

(b) $$\text{Avg.} = \frac{1}{14-6}\int_6^{14}\left[80 - 10\cos\left(\frac{\pi t}{12}\right)\right] dt$$

$$= \frac{1}{8}(697.2957795)$$

$$= 87.162 \quad \text{or} \quad 87.161$$

$$\approx 87^\circ \text{ F}$$

3
- 2: integral
  - 1: limits and $1/(14-6)$
  - 1: integrand
- 1: answer
  - 0/1 if integral not of the form $\displaystyle\frac{1}{b-a}\int_a^b F(t)\,dt$

(c) $$\left[80 - 10\cos\left(\frac{\pi t}{12}\right)\right] - 78 \ge 0$$

$$2 - 10\cos\left(\frac{\pi t}{12}\right) \ge 0$$

$$\left.\begin{matrix} 5.230 \\ \text{or} \\ 5.231 \end{matrix}\right\} \le t \le \left\{\begin{matrix} 18.769 \\ \text{or} \\ 18.770 \end{matrix}\right.$$

2
- 1: inequality or equation
- 1: solutions with interval

(d) $$C = 0.05\int_{\substack{5.231 \\ \text{or} \\ 5.230}}^{\substack{18.770 \\ \text{or} \\ 18.769}} \left(\left[80 - 10\cos\left(\frac{\pi t}{12}\right)\right] - 78\right) dt$$

$$= 0.05(101.92741) = 5.096 \approx \$5.10$$

3
- 2: integral
  - 1: limits and 0.05
  - 1: integrand
- 1: answer
  - 0/1 if integral not of the form $\displaystyle k\int_a^b (F(t) - 78)\,dt$

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6. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.
   (a) Show that $\dfrac{dy}{dx} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$.
   (b) Write an equation of each horizontal tangent line to the curve.
   (c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x– and y–coordinates of point P.

(a) $6y^2\dfrac{dy}{dx} + 6x^2\dfrac{dy}{dx} + 12xy - 24x + 6\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}(6y^2 + 6x^2 + 6) = 24x - 12xy$

$\dfrac{dy}{dx} = \dfrac{24x - 12xy}{6x^2 + 6y^2 + 6} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$

2 $\begin{cases} 1: \text{ implicit differentiation} \\ 1: \text{ verifies expression for } \dfrac{dy}{dx} \end{cases}$

(b) $\dfrac{dy}{dx} = 0$

$4x - 2xy = 2x(2 - y) = 0$

$x = 0$ or $y = 2$

When $x = 0$, $2y^3 + 6y = 1$; $y = 0.165$

There is no point on the curve with y coordinate of 2.

$y = 0.165$ is the equation of the only horizontal tangent line.

4 $\begin{cases} 1: \text{ sets } \dfrac{dy}{dx} = 0 \\ 1: \text{ solves } \dfrac{dy}{dx} = 0 \\ 1: \text{ uses solutions for } x \text{ to find equations of horizontal tangent lines} \\ 1: \text{ verifies which solutions for } y \text{ yield equations of horizontal tangent lines} \end{cases}$

Note: max 1/4 [1-0-0-0] if $dy/dx = 0$ is not of the form $g(x, y)/h(x, y) = 0$ with solutions for both x and y

(c) $y = -x$ is equation of the line.

$2(-x)^3 + 6x^2(-x) - 12x^2 + 6(-x) = 1$

$-8x^3 - 12x^2 - 6x - 1 = 0$

$x = -1/2, \quad y = 1/2$

–or–

$\dfrac{dy}{dx} = -1$

$4x - 2xy = -x^2 - y^2 - 1$

$4x + 2x^2 = -x^2 - x^2 - 1$

$4x^2 + 4x + 1 = 0$

$x = -1/2, \quad y = 1/2$

3 $\begin{cases} 1: \ y = -x \\ 1: \text{ substitutes } y = -x \text{ into equation of curve} \\ 1: \text{ solves for } x \text{ and } y \end{cases}$

–or–

3 $\begin{cases} 1: \text{ sets } \dfrac{dy}{dx} = -1 \\ 1: \text{ substitutes } y = -x \text{ into } \dfrac{dy}{dx} \\ 1: \text{ solves for } x \text{ and } y \end{cases}$

Note: max 2/3 [1-1-0] if importing incorrect derivative from part (a)